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RESEARCH STUDY ON STABILIZATION AND CONTROL

MODERN SAMPLED-DATA CONTROL THEORY

SYSTEMS RESEARCH LABORATORY

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PREPARED FOR GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

BI-MONTHLY REPORT

RESEARCH STUDY ON STABILIZATION AND CONTROL

– MODERN SAMPLED-DATA CONTROL THEORY

DESIGN OF THE

SUBTITLE: LARGE SPACE TELESCOPE

SYSTEM

September 10, 1974 NAS8-29853

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1. Prediction by Numerical Methods of Self-Sustained Oscillations in a Two-Axis Model of the Nonlinear Continuous-Data LST System

1-1. Introduction

It has been demonstrated in [1] that the methods of continuous and discrete describing function analysis can be applied to predict the existence of self-sustained oscillations in the single-axis model of the LST system with nonlinear CMG friction characteristics. Furthermore, it has been shown in [2] that the stability equations as a result of the describing function analysis may be solved by a numerical-iterative technique instead of the usual graphical methods. With an appropriate guess of the initial condition, the numerical method is found to be quite effective in leading to a convergent solution rapidly.

In the two-axis model of the LST system, the system contains two nonlinearities, and the general form of the stability equation in the continuous-data case is

$$1 + \hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A) = 0 \qquad (1\text{-}1)$$

where $\hat{G}_B(j\omega)$ and $\hat{G}_C(j\omega)$ denote transfer functions which depend on the linear elements of the coupled systems, and N(A) represents the describing function of the CMG nonlinearity. In general, $\hat{G}_B(j\omega)$ and $\hat{G}_C(j\omega)$ are functions of frequency ω, and N(A) is a function of the amplitude A of the assumed sinusoidal input to the nonlinearity.

The usual graphical method cannot be used to solve for the values

of ω and A for self-sustained oscillations in Eq. (1-1), due to the $N^2(A)$ term in the equation. However, if the system parameters are such that the linear term $\hat{G}_B(j\omega)N(A)$ dominates over the quadratic term, or vice versa, then by neglecting the smaller quantity in Eq. (1-1), an approximate solution may still be obtained graphically.

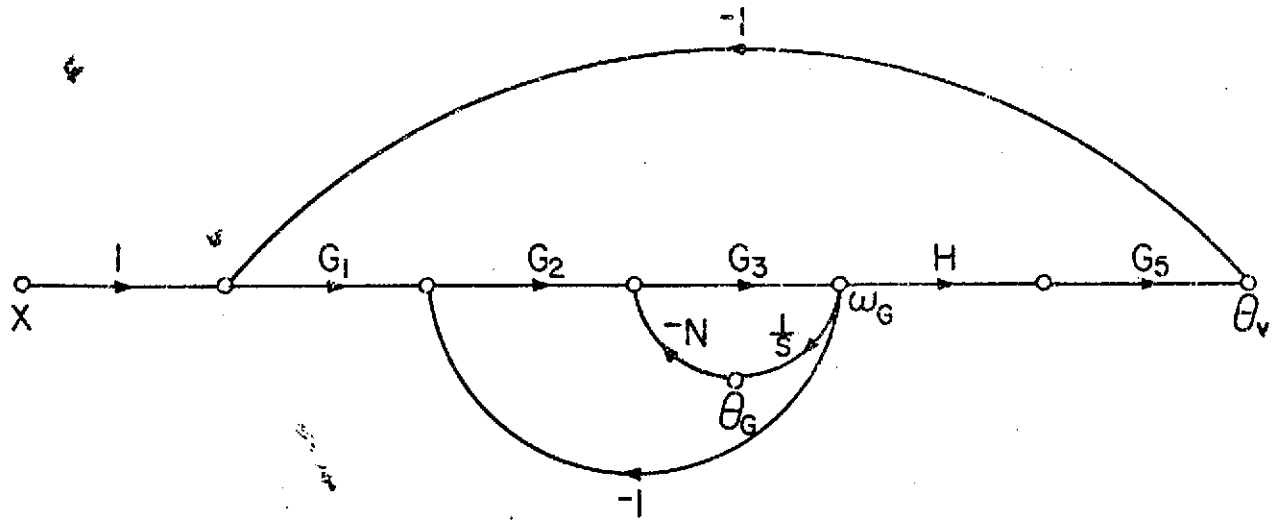
A direct but more time consuming approach of solving Eq. (1-1) would be to calculate the terms in Eq. (1-1) for a wide range of values of ω and A until all combinations which satisfy the equation are found.

Still a third alternative of solving Eq. (1-1) is to use the numerical-iterative method reported in [2]. If an appropriate initial guess can be made, the numerical-iterative method should lead to the exact solutions in an efficient manner.

All three of the above-mentioned methods have been applied to the two-axis continuous-data LST system. It is found that the graphical method with approximation and the direct method can provide useful information to the final solutions, and the numerical-iterative method is effective in arriving at the exact solution on a digital computer. The results of these studies are reported in the ensueing sections.

1-2. The Continuous-Data Two-Axis LST System Model and Its Stability Equation

Figure 1-1 Shows the signal flow graph representation of the continuous-data single-axis LST system. The nonlinear friction of the CMG is modeled by the branch with the gain N. If two such models are coupled together at the output stage through output torsional coupling, the two-axis model of Fig. 1-2 results. This representation of a continuous-data two axis LST system may not be a rigorous one from the



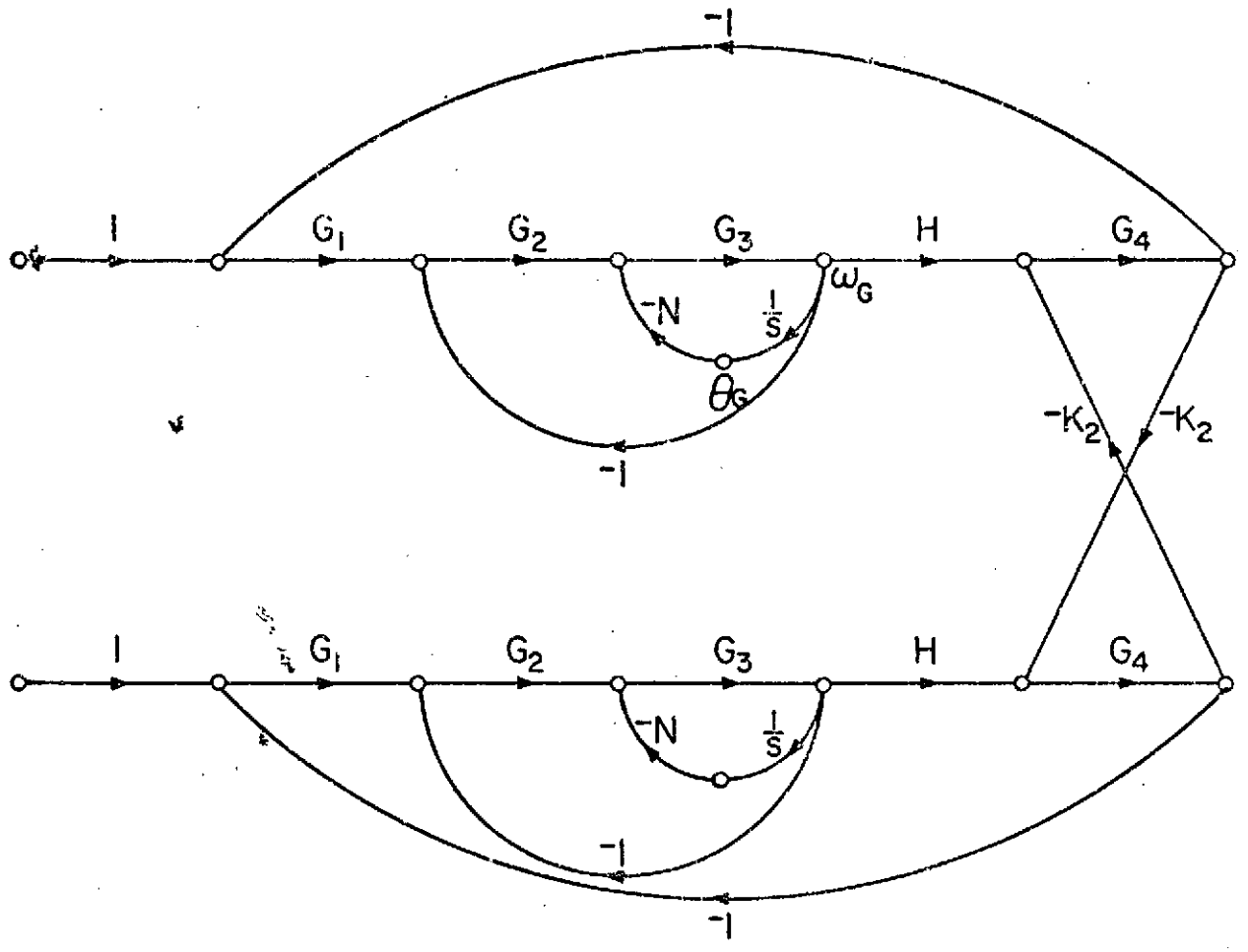
$$G_1 = (K_o + K_1 s)\,\frac{K_I}{(K_p s + K_I)}$$

$$G_2 = \frac{K_p s + K_I}{s}$$

$$G_3 = \frac{1}{J_G s}$$

$$G_5 = \frac{1}{J_V s^2}$$

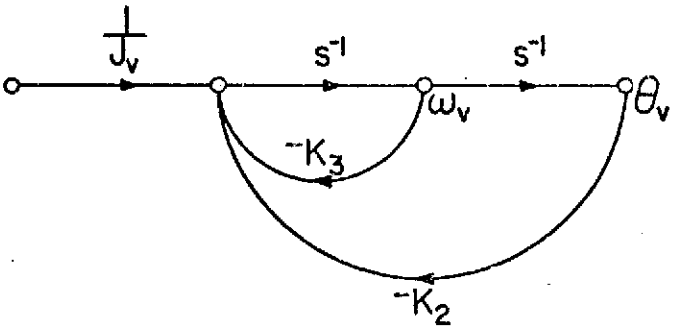
Figure 1-1. The simplified single-axis continuous-data LST system model.



$$G_1 = \frac{(K_0+K_1 s)K_I}{K_p s + K_I}$$

$$G_2 = \frac{K_p s + K_I}{s}$$

$$G_3 = \frac{1}{J_G s}$$



$$\left| \leftarrow \quad G_4 = \frac{1/J_v}{s^2 + K_3 s + K_2} \quad \rightarrow \right|$$

Figure 1-2. The simplified two-axis continuous-data LST system model.

structural standpoint. However, the purpose of the present study is to develop analytical techniques which are applicable to multi-coupled nonlinear systems. It is conjectured that for a two-axis nonlinear system, the describing function analysis will generally lead to a stability equation of the form of Eq. (1-1). Therefore, for these above-mentioned reasons, the model shown in Fig. 1-2 is considered to be adequate. The techniques developed in this report can be applied to the prediction of self-sustained oscillations in any continuous-data two-axis nonlinear system that is amenable to the describing function method.

With reference to Fig. 1-2, it can be seen that there are seven individual loops, with the following loop gains:

$$A_1 = -\frac{NG_3}{s} \tag{1-2}$$

$$A_2 = -\frac{NG_3}{s} \tag{1-3}$$

$$B_1 = -G_2G_3 \tag{1-4}$$

$$B_2 = -G_2G_3 \tag{1-5}$$

$$C_1 = -G_1G_2G_3G_4H \tag{1-6}$$

$$C_2 = -G_1G_2G_3G_4H \tag{1-7}$$

$$D = K_2^2G_4^2 \tag{1-8}$$

where G_1, G_2, G_3, and G_4 are transfer functions as defined in Fig. 1-2. All system parameters and variables, including the nonlinear describing

function N, are consistent with those defined in the single-axis LST model reported previously [1,2]. The describing function N(A) is a function of the amplitude of the input sinusoid of the CMG nonlinearities. In the present case, since the two axes are identical, and the couplings are symmetric, it is assumed that the amplitudes of the input signals at the two nonlinearities are identical.

The characteristic equation of the coupled system in Fig. 1-2 is

$$\Delta = 0 \tag{1-9}$$

where

$$\begin{aligned}\Delta = 1 &- (A_1 + A_2 + B_1 + B_2 + C_1 + C_2 + D) \\ &+ (A_1A_2 + A_1B_2 + A_1C_2 + A_1D + B_1A_2 + B_1B_2 + B_1C_2 + B_1D \\ &\quad + C_1A_2 + C_1B_2 + C_1C_2 + A_2D + B_2D) \\ &- (A_1A_2D + A_1B_2D + B_1A_2D + B_1B_2D)\end{aligned} \tag{1-10}$$

The last equation is simplified if we define

$$\begin{aligned}A &= A_1 = A_2 \\ B &= B_1 = B_2 \\ C &= C_1 = C_2\end{aligned} \tag{1-11}$$

Then, Eq. (1-10) becomes

$$\begin{aligned}\Delta = 1 &- 2(A + B + C) - D + (A^2 + B^2 + C^2 + 2AB + 2AC + 2BC \\ &+ 2AD + 2BD) - (A + B)^2D\end{aligned} \tag{1-12}$$

or

$$\Delta = 1 - 2(A + B + C) - D + (A + B + C)^2 - (A + B)^2 D \tag{1-13}$$

Substituting Eqs. (1-2) through (1-8) in Eq. (1-13) yields

$$\begin{aligned}\Delta = 1 &+ \frac{2G_3N}{s} + 2G_2G_3 + 2G_1G_2G_3G_4H - K_2^2G_4^2 \\ &+ \frac{G_3^2}{s^2}N^2 + G_2^2G_3^2 + (G_1G_2G_3G_4H)^2 + \frac{2NG_2G_3^2}{s} \\ &+ \frac{2NG_1G_2G_3^2G_4H}{s} + 2G_1G_2^2G_3^2G_4H - \frac{2NG_3G_4^2K_2^2}{s} \\ &- 2G_2G_3G_4^2K_2^2 - \frac{N^2G_3^2G_4^2K_2^2}{s^2} - \frac{2NG_2G_3^2G_4^2K_2^2}{s} \\ &- G_2^2G_3^2G_4^2K_2^2\end{aligned} \tag{1-14}$$

The characteristic equation in Eq. (1-9) can be written as

$$\Delta = 1 + G_A + G_BN + G_CN^2 = 0 \tag{1-15}$$

where

$$\begin{aligned}G_A = &\ 2G_2G_3 + 2G_1G_2G_3G_4H - K_2^2G_4^2 + G_2^2G_3^2 + (G_1G_2G_3G_4H)^2 \\ &+ 2G_1G_2^2G_3^2G_4H - 2G_2G_3G_4^2K_2^2 - G_2^2G_3^2G_4^2K_2^2\end{aligned} \tag{1-16}$$

$$\begin{aligned}G_B = &\ \frac{2G_3}{s} + \frac{2G_2G_3^2}{s} + \frac{2G_1G_2G_3^2G_4H}{s} - \frac{2G_3G_4^2K_2^2}{s} \\ &- \frac{2G_2G_3^2G_4^2K_2^2}{s}\end{aligned} \tag{1-17}$$

$$G_C = \frac{G_3^2}{s^2} - \frac{G_3^2 G_4^2 K_2^2}{s^2} \tag{1-18}$$

For stability analysis, the characteristic equation of Eq. (1-15) is written as

$$1 + \hat{G}_B N + \hat{G}_C N^2 = 0 \tag{1-19}$$

where

$$\hat{G}_B = \frac{G_B}{1 + G_A} \tag{1-20}$$

$$\hat{G}_C = \frac{G_C}{1 + G_A} \tag{1-21}$$

Equation (1-19) is a function of the system frequency ω and the input amplitude A. Since this equation is defined in the complex plane, it represents a set of two nonlinear equations with two unknowns. When these nonlinear equations have a solution for ω and A, it represents a condition of self-sustained oscillations for the system. The oscillations may be stable or unstable; therefore, the solutions must always be checked for stability.

1-3. Prediction of Self-Sustained Oscillations By The Approximation Method

It was mentioned in Sec. 1-1 that the graphical method of predicting self-sustained oscillations can still be applied to the two-axis LST system if one of the last two terms in Eq. (1-1) can be neglected. In

other words, the following two conditions may exist:

1. $|\hat{G}_B(j\omega)N(A)| >> |\hat{G}_C(j\omega)N^2(A)|$

Then Eq. (1-1) may be approximated by

$$1 + \hat{G}_B(j\omega)N(A) = 0 \tag{1-22}$$

and the condition of self-sustained oscillations is found from the following equation:

$$\hat{G}_B(j\omega) = -\frac{1}{N(A)} \tag{1-23}$$

2. $|\hat{G}_B(j\omega)N(A)| << |\hat{G}_C(j\omega)N^2(A)|$

Then Eq. (1-1) is approximated by

$$1 + \hat{G}_C(j\omega)N^2(A) = 0 \tag{1-24}$$

and the condition of self-sustained oscillations is found from

$$\hat{G}_C(j\omega) = -\frac{1}{N^2(A)} \tag{1-25}$$

Therefore, the graphical solutions involve the plotting of the curves for $\hat{G}_B(j\omega)$ and $-1/N(A)$, or $\hat{G}_C(j\omega)$ and $-1/N^2(A)$, as the case may be.

Figure 1-3 illustrates the plots of $|\hat{G}_B(j\omega)N(A)|$ and $|\hat{G}_C(j\omega)N^2(A)|$ versus ω for various values of A, and $|N(A)|$ versus A. The following parameters are used for the LST system:

$$J_V = 10^5 , \quad J_G = 2.1 , \quad K_p = 216 , \quad K_I = 9700$$

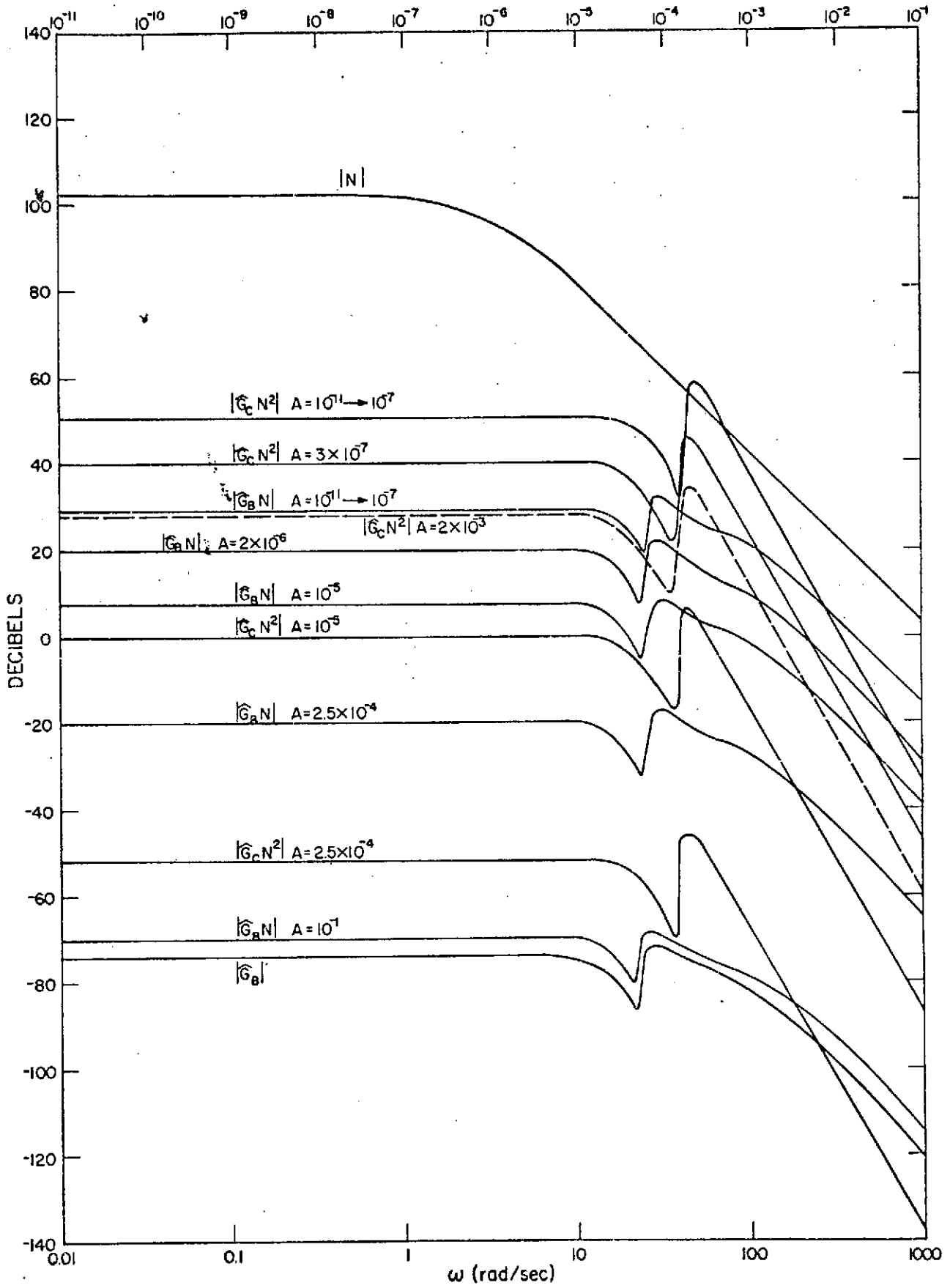

10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
140
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
DECIBELS
|N|
$|\hat{G}_c N^2|$ A = $10^{-11} \rightarrow 10^{-7}$
$|\hat{G}_c N^2|$ A = 3×10^{-7}
$|\hat{G}_B N|$ A = $10^{-11} \rightarrow 10^{-7}$
$|\hat{G}_c N^2|$ A = 2×10^{-3}
$|\hat{G}_B N|$ A = 2×10^{-6}
$|\hat{G}_B N|$ A = 10^{-5}
$|\hat{G}_c N^2|$ A = 10^{-5}
$|\hat{G}_B N|$ A = 2.5×10^{-4}
$|\hat{G}_c N^2|$ A = 2.5×10^{-4}
$|\hat{G}_B N|$ A = 10^{-1}
$|\hat{G}_B|$
0.01
0.1
1
10
100
1000
ω (rad/sec)

Figure 1-3.

$H = 600$, $K_0 = 5758.35$, $K_1 = 1371.02$, $K_2 = 100$, $K_3 = 3$

$\gamma = 1.38 \times 10^7$

The last two parameters, K_2 and K_3 are the coefficients of coupling between the two axes.

The curves in Fig. 1-3 give information on the ranges of ω and A in which the approximation of Eq. (1-22) or Eq. (1-24) is valid. Figure 1-4 illustrates the regions in the ω versus A plane in which the two approximations are valid. The cross-hatched area represents the region in which no approximation can be made, and the graphical method cannot be used. The criterion of $\geq$20 db and $\leq$-20 db is used for magnitude comparison for significance.

The results of Fig. 1-4 show that the graphical approximation method is valid for the following ranges of ω and A:

1. Use $1 + \hat{G}_C N^2 = 0$:

$$A \leq 10^{-7} \qquad \omega \leq 40 \text{ rad/sec}$$

$$\text{and} \quad 60 \leq \omega \leq 80 \text{ rad/sec}$$

2. Use $1 + \hat{G}_B N = 0$:

$$A \geq 6 \times 10^{-5} \qquad \omega \geq 600 \text{ rad/sec.}$$

For the region of validity of case 1 above, Fig. 1-5 shows the plots of $\hat{G}_C$ and $-1/N^2$. The heavy portions of the curves indicate the parts which are valid for stability analysis. Similarly, Fig. 1-6 shows the case when the equation $1 + \hat{G}_B N = 0$ may be used for approximation. Again, the intersection between the heavy portions of the $\hat{G}_B$ and the $-1/N$ curves would indicate the possibility of self-sustained oscillation in the two-axis LST system. Since there are no intersections

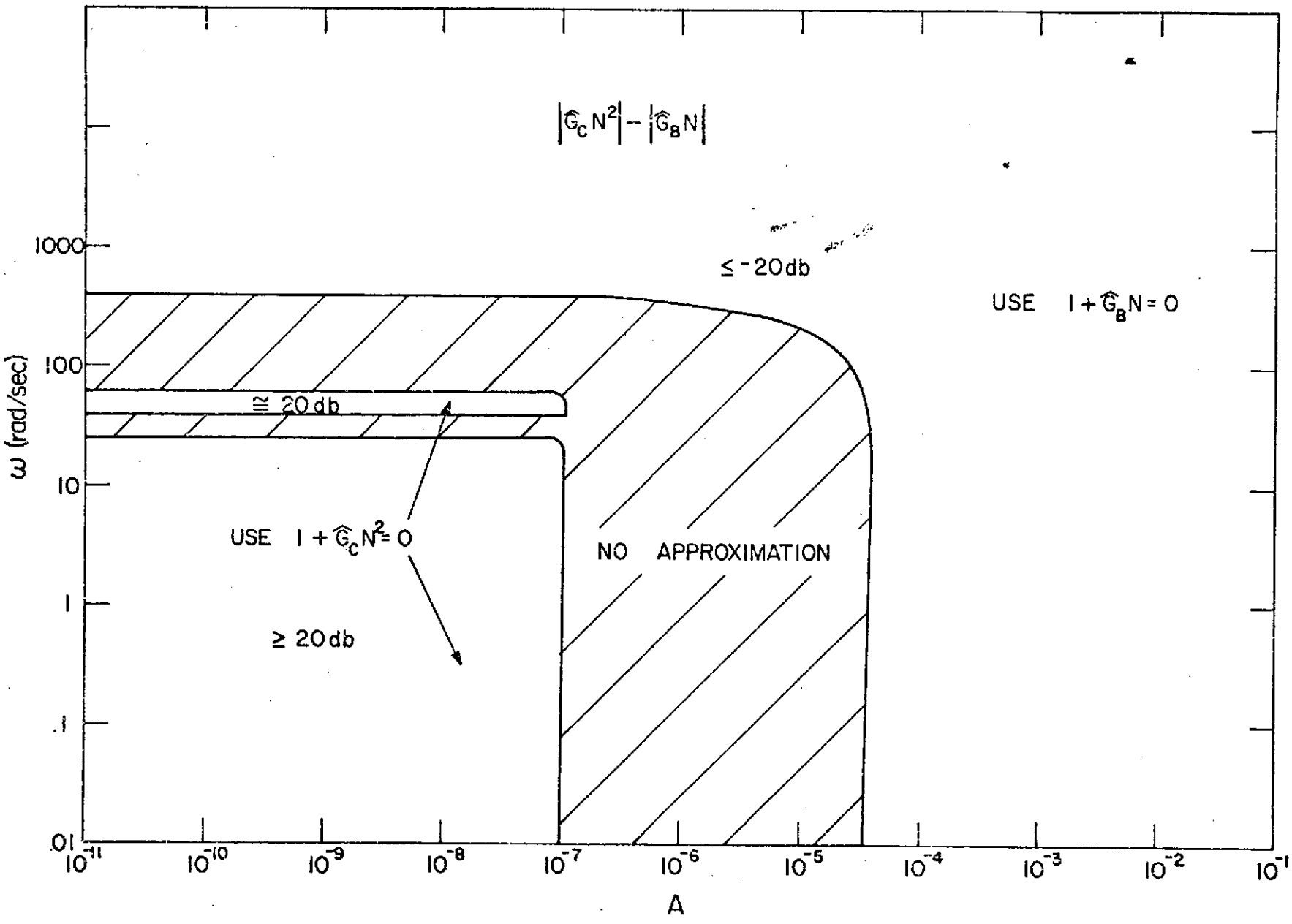

$|\hat{G}_C N^2| - |\hat{G}_B N|$
1000
100
10
1
.1
.01
ω (rad/sec)
≤ -20db
USE $1 + \hat{G}_B N = 0$
≅ 20db
USE $1 + \hat{G}_C N^2 = 0$
NO APPROXIMATION
≥ 20db
10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
A

Figure 1-4.

between the valid curves in Figs. 1-5 and 1-6, the system under study is stable for the parameter values used. This result will be substantiated by the two other methods which are discussed in the following sections.

1-4. A Direct Method of Predicting Self-Sustained Oscillations in The Two-Axis LST System

A direct but time-consuming method of predicting self-sustained oscillations in the two-axis continuous-data LST system is to calculate all the terms in the following stability equation for a wide range of values of ω and A:

$$1 + \hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A) = 0 \qquad (1\text{-}26)$$

Although the direct method would be quite tedious, and it is possible that with a given selected increment of variation for the values of ω and A, the solution runs may still miss the exact solutions, the results in general will give insight to the appropriate guess on the initial values for the numerical-iteration method. Therefore, it is enlightening to investigate this direct approach at this point.

Figures 1-7 through 1-9 show the plots of $\hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A)$ for $\gamma = 1.38 \times 10^7$, and all the system parameters as given previously, and for three different sets of values for the coupling coefficients K_2 and K_3. Note that for the three cases illustrated, all the trajectories for various combinations of A and ω do not intersect the critical point which is at 0 db and -180 degrees. Thus the system is stable. However, the results show that as the value of K_2 is decreased, the trajectories

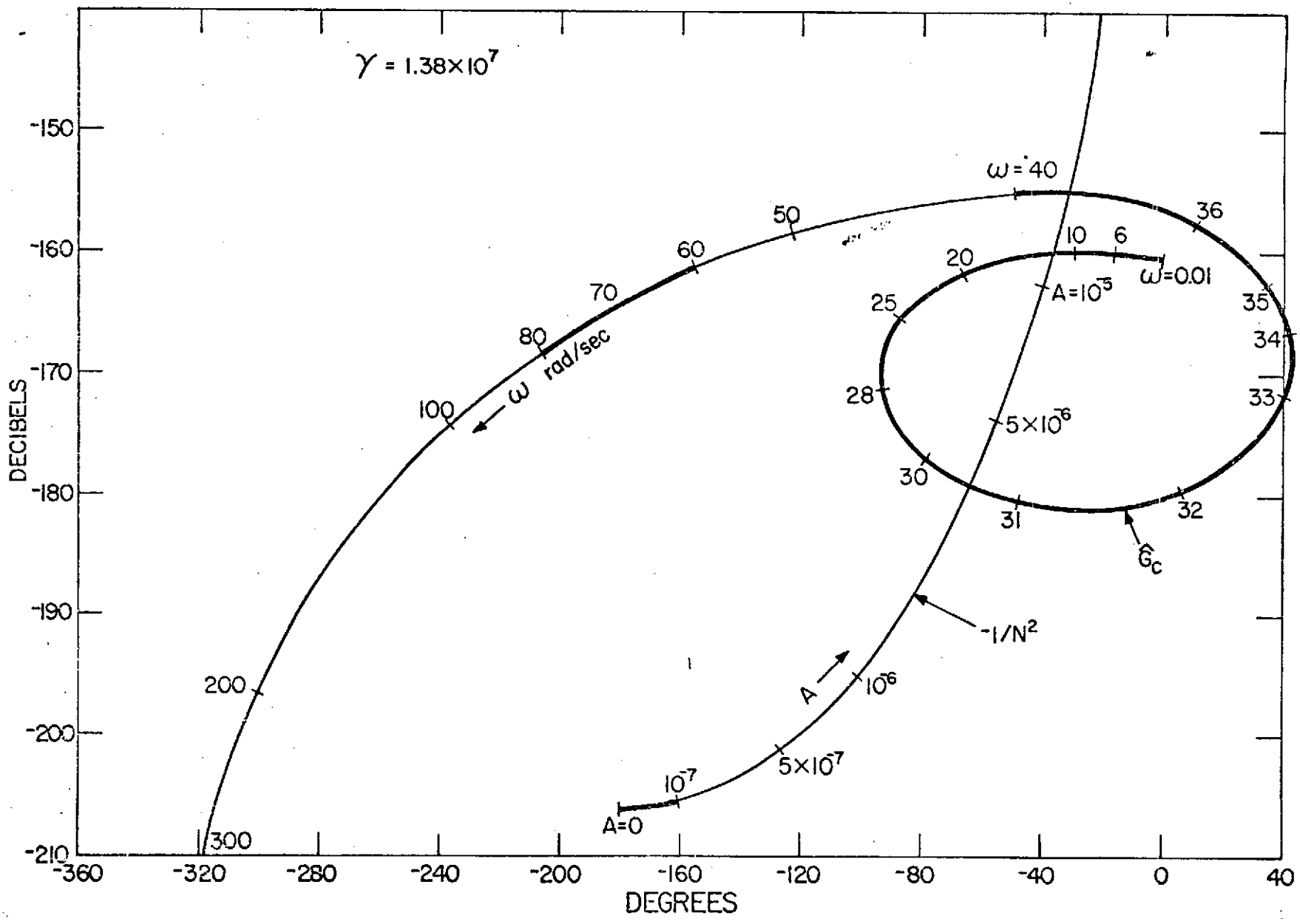

$\gamma = 1.38\times10^{7}$
-150
-160
-170
-180
-190
-200
-210
DECIBELS
-360
-320
-280
-240
-200
-160
-120
-80
-40
0
40
DEGREES
ω = 40
50
60
70
80
100
200
300
ω rad/sec
36
10
6
20
25
28
30
31
32
33
35
34
ω=0.01
A=10⁻³
5×10⁻⁶
Gc
-1/N²
A
10⁶
5×10⁻⁷
10⁻⁷
A=0

Figure 1-5.

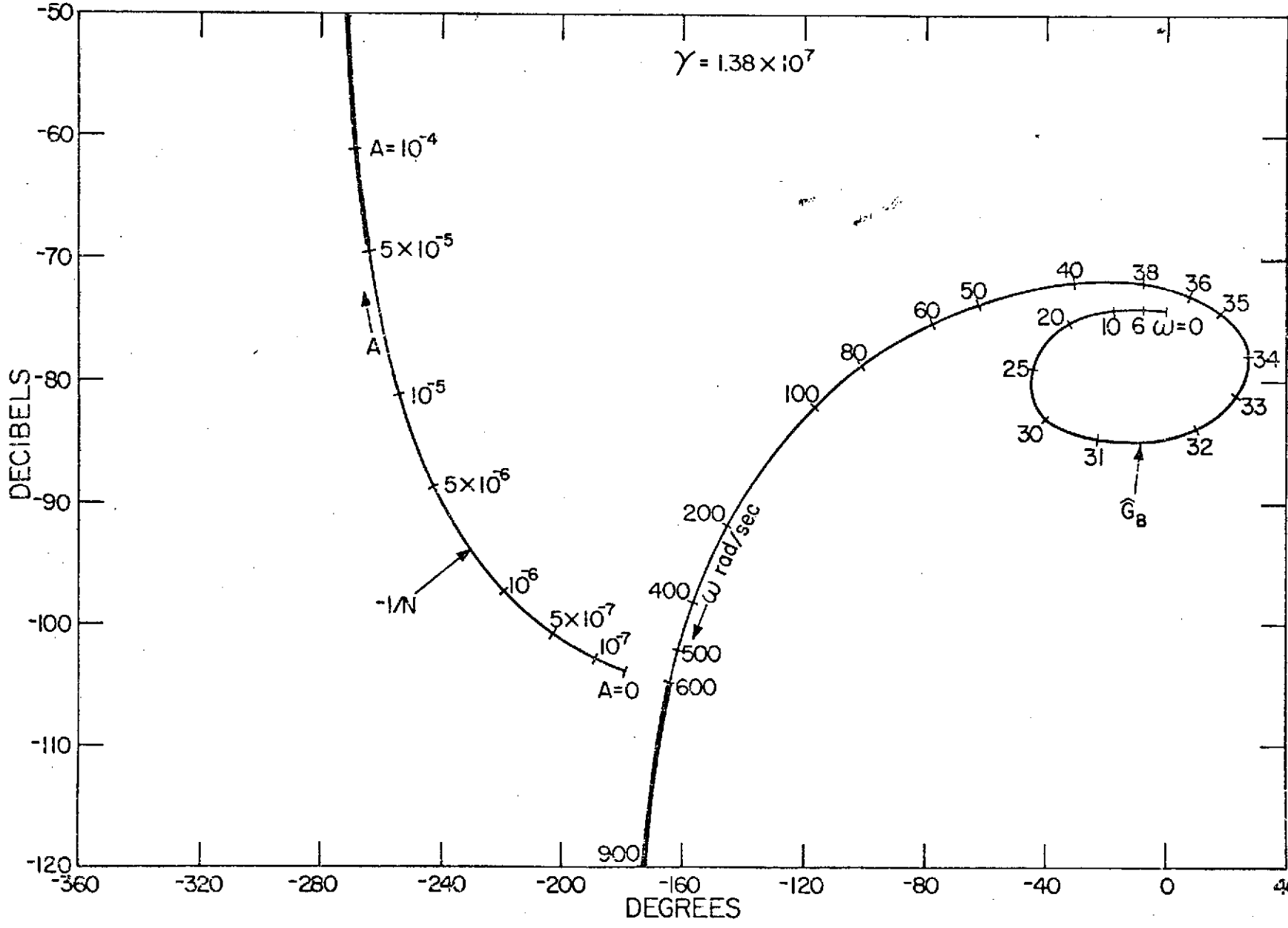

$\gamma = 1.38 \times 10^7$
$A = 10^{-4}$
5×10^{-5}
A
10^{-5}
5×10^{-6}
10^{-6}
5×10^{-7}
10^{-7}
A=0
-1/N
ω rad/sec
900
$\hat{G}_B$
DECIBELS
DEGREES
-50
-60
-70
-80
-90
-100
-110
-120
-360
-320
-280
-240
-200
-160
-120
-80
-40
0
40
100
200
400
500
600
80
60
50
40
38
36
35
34
33
32
31
30
25
20
10
6
ω=0

Figure 1-6.

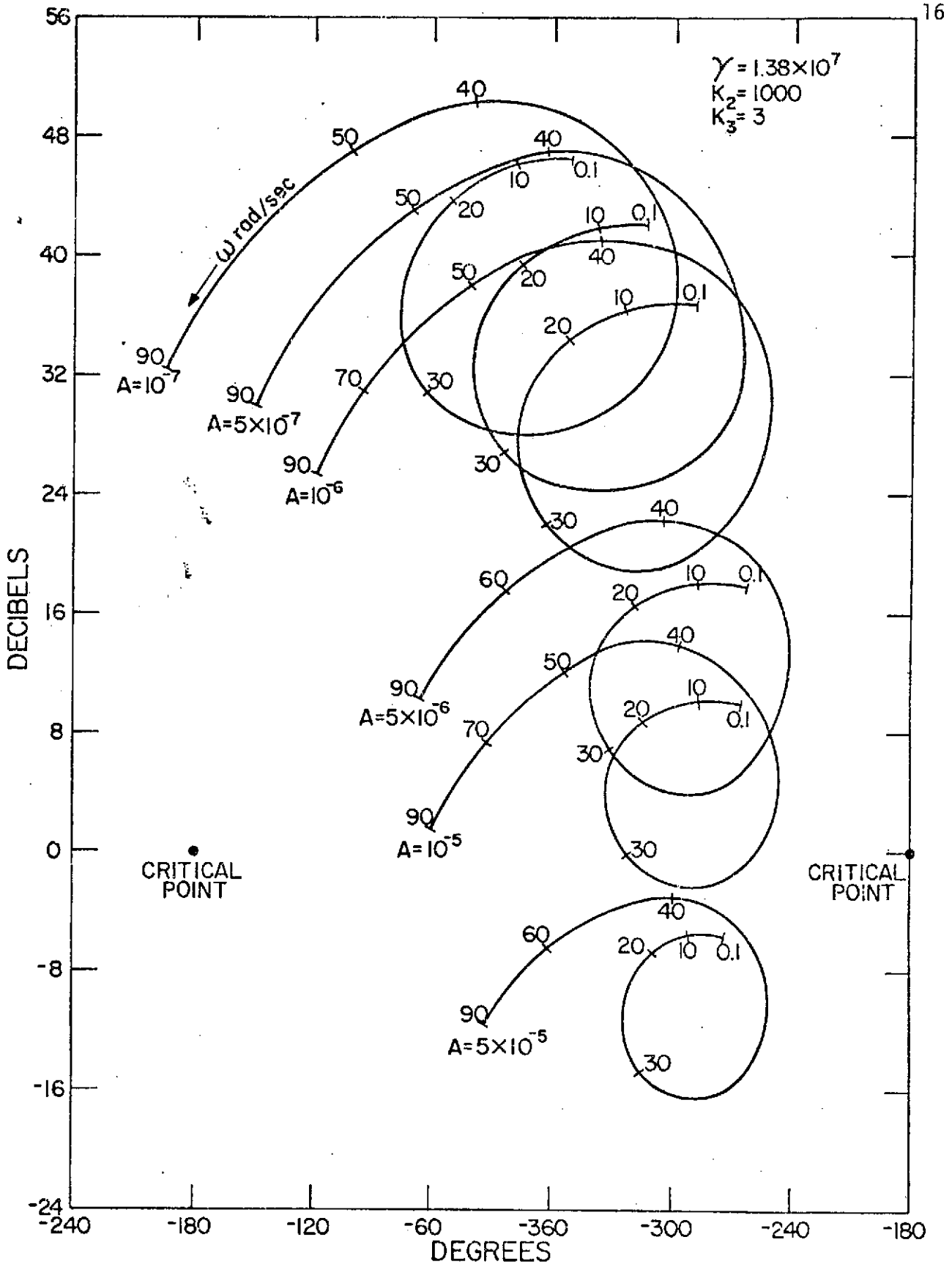

γ = 1.38×10^7
K2 = 1000
K3 = 3
ω rad/sec
A=10^-7
A=5×10^-7
A=10^-6
A=5×10^-6
A=10^-5
A=5×10^-5
CRITICAL POINT
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
-240
-180
-120
-60
-360
-300
-240
-180

Figure 1-7.

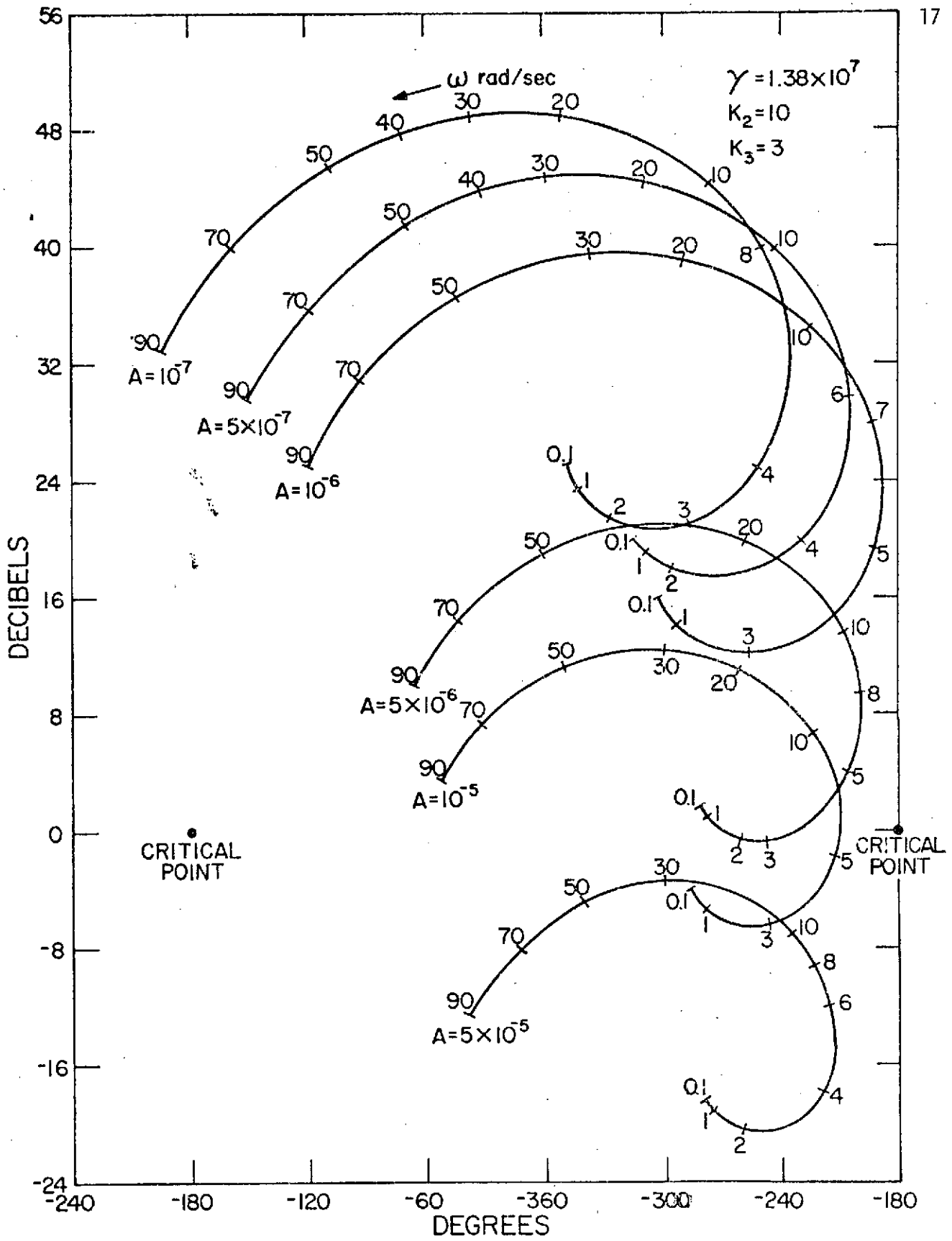

ω rad/sec
γ = 1.38×10^7
K2 = 10
K3 = 3
A = 10^-7
A = 5×10^-7
A = 10^-6
A = 5×10^-6
A = 10^-5
A = 5×10^-5
CRITICAL POINT
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
-240
-180
-120
-60
-360
-300
-240
-180

Figure 1-8.

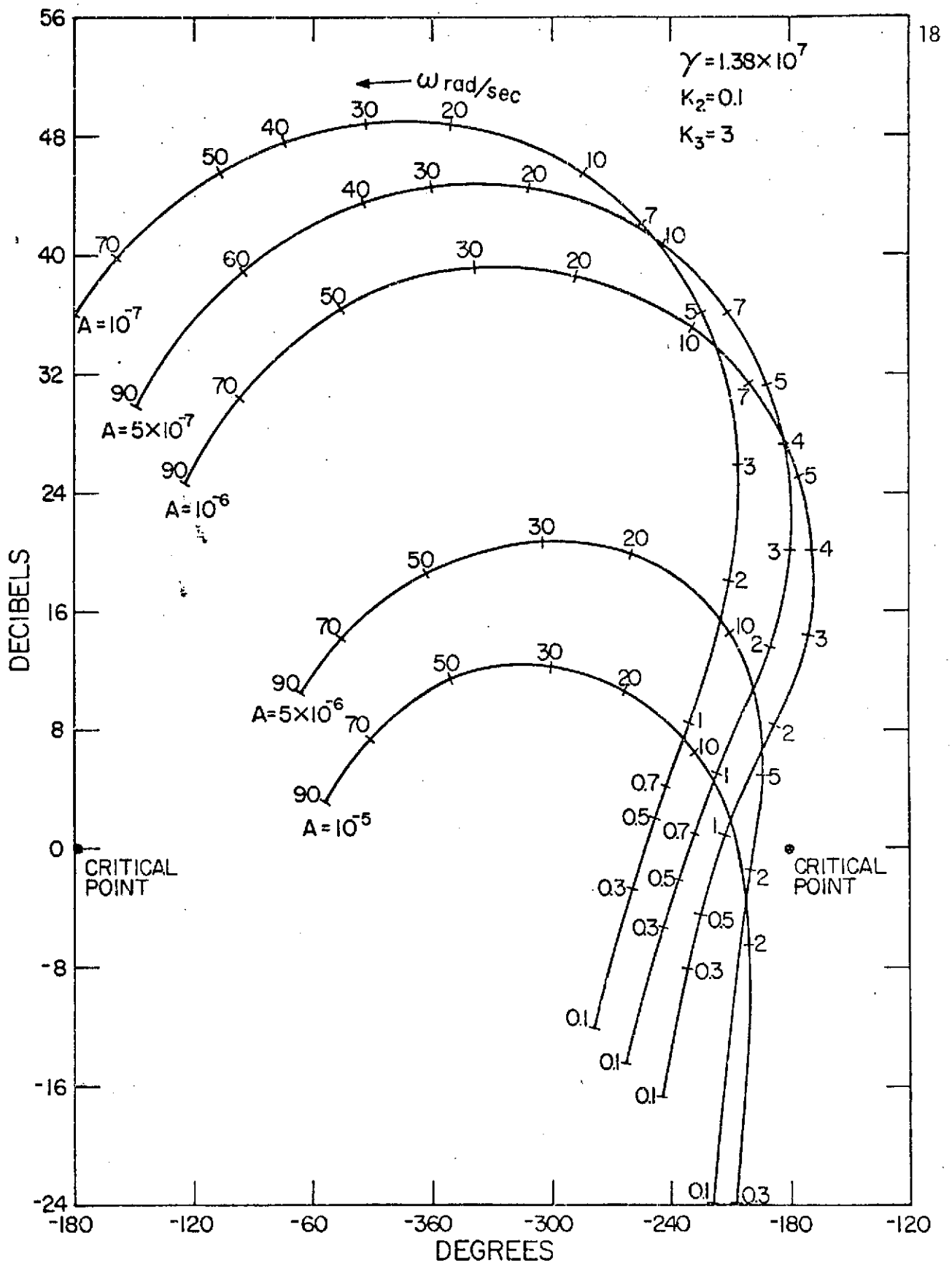

$\gamma = 1.38 \times 10^7$
$K_2 = 0.1$
$K_3 = 3$
ω rad/sec
$A = 10^{-7}$
$A = 5 \times 10^{-7}$
$A = 10^{-6}$
$A = 5 \times 10^{-6}$
$A = 10^{-5}$
CRITICAL POINT
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
-180
-120
-60
-360
-300
-240
-180
-120

Figure 1-9.

get closer to the critical point. Figure 1-10 shows the trajectories when $K_2 = 0.1$ and $K_3 = 0.1$. It is noticed that several trajectories are very close to the critical point. Figure 1-11 shows a magnified version of Fig. 1-10 around the critical point. The figure shows that the trajectories for $A = 5 \times 10^{-6}$, 10^{-6}, 5×10^{-6}, are all very close to the critical point. This gives indications that there may be more than one solution. It will be shown in the next section by the numerical-iterative method that there are indeed two solutions for the stability equation. One is at $A = 5.9867 \times 10^{-7}$ and $\omega = 1.88$ rad/sec, and the other is at $A = 5.07397 \times 10^{-6}$ and $\omega = 4.1086$ rad/sec. These solutions must still be checked for stable or unstable equilibrium solutions.

1-5. Exact Solution of the Stability Equation by Numerical-Iterative Techniques

In this section the stability equation developed in Sec. 1-2 is solved numerically for its exact solutions. The numerical method utilized has been described in [2] and is found to be quite effective for the two-axis LST system.

The stability equation, Eq. (1-19), can be written as

$$1 + \hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A) = 0 \tag{1-27}$$

Define

$$\hat{G}_B(j\omega) = G_{R1} + jG_{I1} \tag{1-28}$$

$$\hat{G}_C(j\omega) = G_{R2} + jG_{I2} \tag{1-29}$$

$$N(A) = N_{R1} + jN_{I1} \tag{1-30}$$

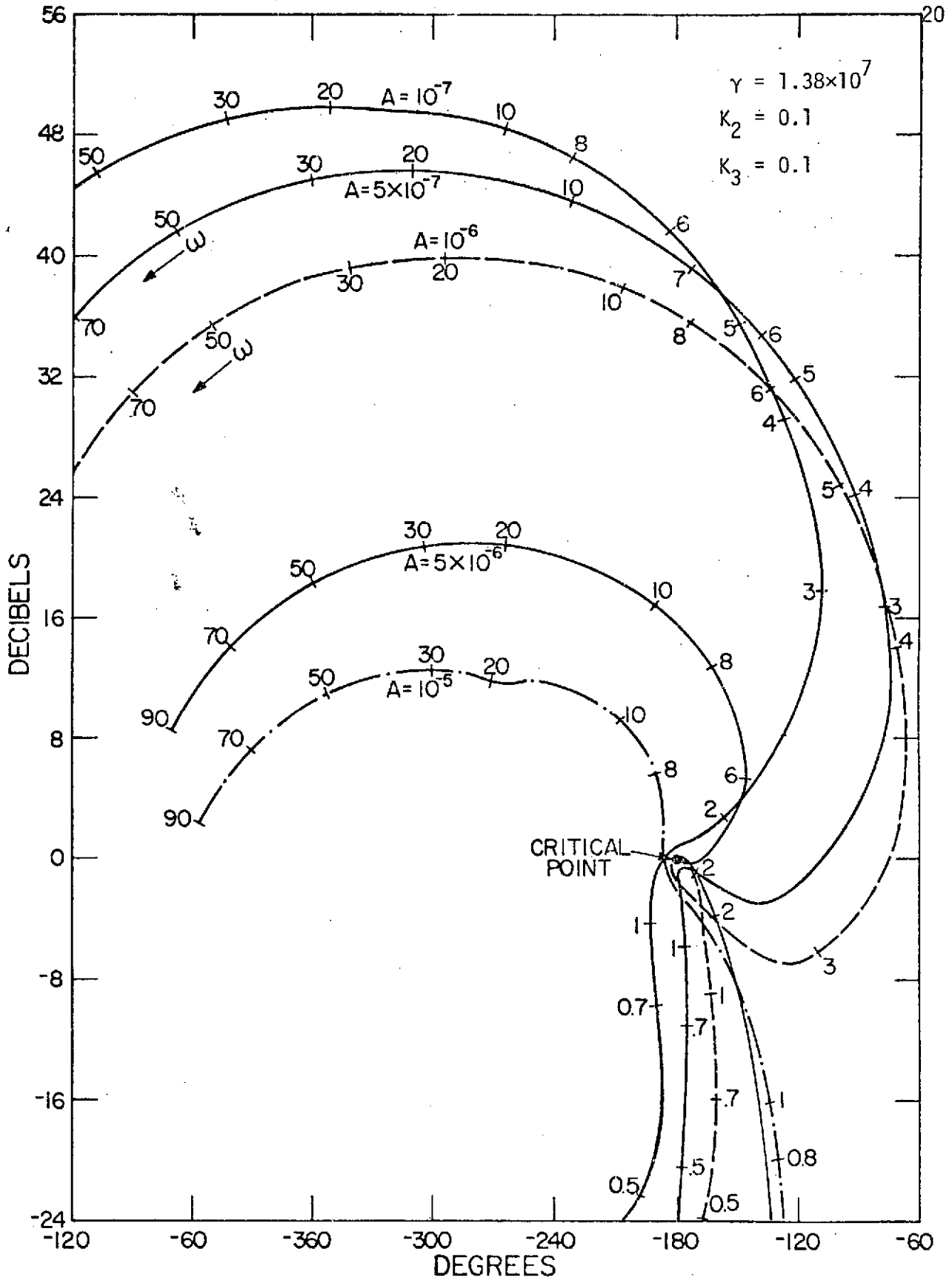

γ = 1.38×10^7
K2 = 0.1
K3 = 0.1
A = 10^-7
A=5×10^-7
A=10^-6
A=5×10^-6
A=10^-5
ω
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
20
-120
-60
-360
-300
-240
-180
-120
-60

Figure 1-10.

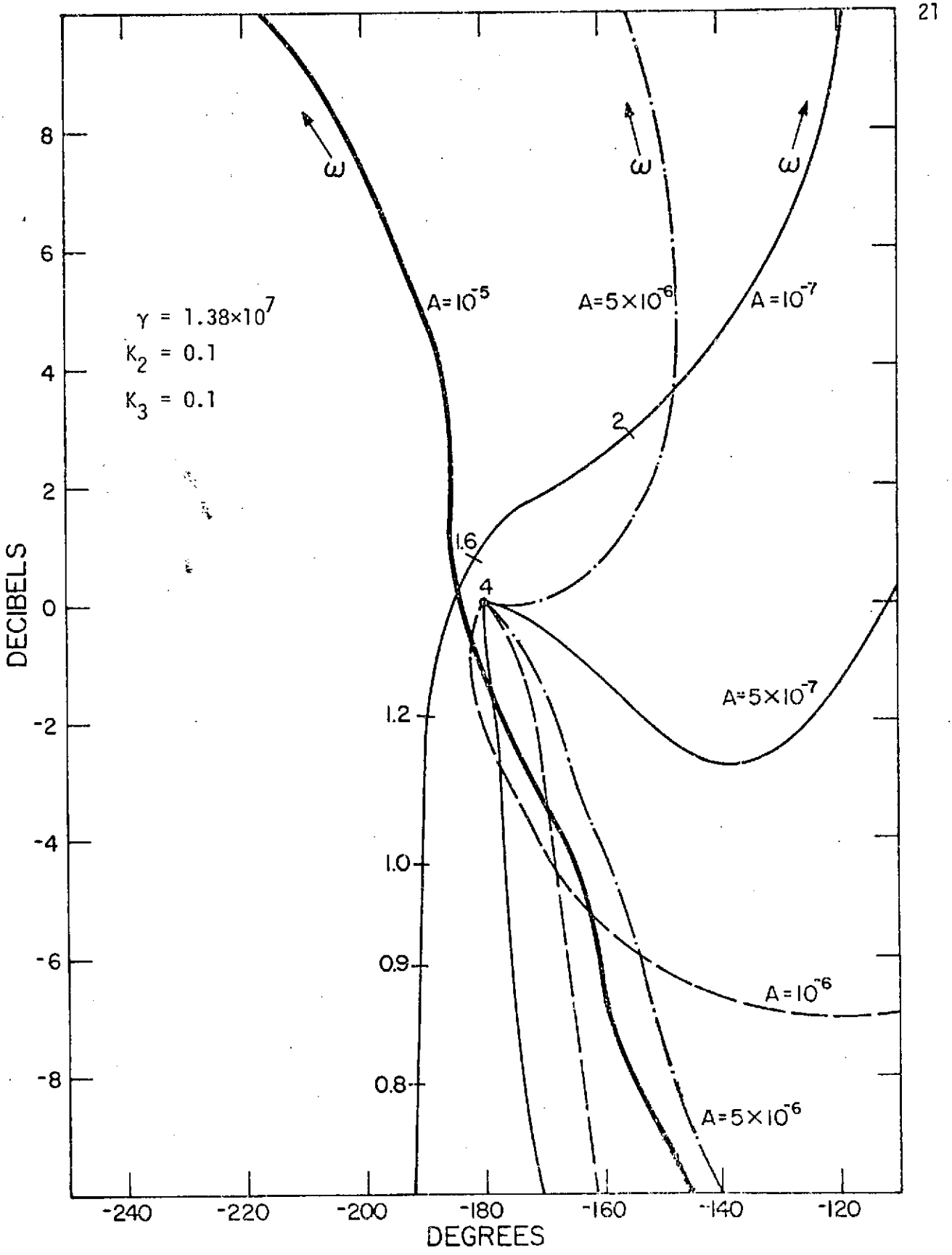

γ = 1.38×10^7
K2 = 0.1
K3 = 0.1
A=10^-5
A=5×10^-6
A=10^-7
A=5×10^-7
A=10^-6
A=5×10^-6
ω
ω
ω
2
1.6
4
1.2
1.0
0.9
0.8
DECIBELS
DEGREES
8
6
4
2
0
-2
-4
-6
-8
-240
-220
-200
-180
-160
-140
-120

Figure 1-11.

$$N^2(A) = N_{R2} + jN_{I2} \tag{1-31}$$

where G_{R1}, G_{I1}, G_{R2}, G_{I2}, N_{R1}, N_{I1}, N_{R2}, and N_{I2} are all real quantities.

If Eqs. (1-28) through (1-31) are substituted into Eq. (1-27), it becomes

$$1 + (G_{R1} + jG_{I1})(N_{R1} + jN_{I1}) + (G_{R2} + jG_{I2})(N_{R2} + jN_{I2}) = 0 \tag{1-32}$$

When the real and imaginary parts are separated, Eq. (1-32) yields

$$\Delta = \Delta_R + j\Delta_I = 0 \tag{1-33}$$

with

$$\Delta_R = 1 + G_{R1}N_{R1} - G_{I1}N_{I1} + G_{R2}N_{R2} - G_{I2}N_{I2} = 0 \tag{1-34}$$

and

$$\Delta_I = G_{I1}N_{R1} + G_{R1}N_{I1} + G_{I2}N_{R2} + G_{R2}N_{I2} = 0 \tag{1-35}$$

Further simplification is possible in Eqs. (1-34) and (1-35) if we recognize that $N_{R2} + jN_{I2} = (N_{R1} + jN_{I1})^2$ but is not necessary since the solution of Eqs. (1-34) and (1-35) will be performed on a digital computer.

Equations (1-33) and (1-34) represent two equations in the two variables ω and A. As before [2], let us define

$$\underline{x} = \begin{pmatrix} \omega \\ A \end{pmatrix} \tag{1-36}$$

$$\underline{F} = \begin{bmatrix} \Delta_R \\ \Delta_I \end{bmatrix} \tag{1-37}$$

Then, Eqs. (1-34) and (1-35) can be written as

$$\underline{F}(\underline{x}) = \underline{0} \tag{1-38}$$

The Newton-type quadratically convergent numerical method described in [2] can now be directly applied to this two-variable system.

To initiate the iterations, an initial solution is needed. The results of the direct calculation method, presented in the previous section provide an adequate initial solution. As in that section, the parameters which need to be varied are K_2 and K_3. Once a solution for a given value of K_2 and K_3 is known, K_2 and/or K_3 could be changed slightly and the new solution is obtained by using the old solution as the initial guess for the new system.

With K_2 and K_3 set to very small values, the system is decoupled, and the solution of the single-axis LST system could be used as an initial guess for that of the two-axis LST system.

If $K_2 = 0.1$ and $K_3 = 0.1$, the results from the direct calculation method shown in Figs. 1-10 and 1-11 indicate that two possible solutions exist. The approximate amplitudes and frequencies of the solutions are

Solution No. 1	$\omega = 5$ rad/sec
	$A = 5 \times 10^{-6}$ rad
Solution No. 2	$\omega = 2$ rad/sec
	$A = 1 \times 10^{-6}$ rad

With these approximate solutions as initial conditions, the results of the numerical iteration procedure are obtained and tabulated in the first set of iterations in Fig. 1-12 and Fig. 1-13. These iterations indicate that the actual solutions are

Solution No. 1 $\omega = 4.1086$ rad/sec

$A = 5.0739 \times 10^{-6}$ rad

Solution No. 2 $\omega = 1.88$ rad/sec

$A = 5.99 \times 10^{-7}$ rad

Figures 1-12 and 1-13 also show the change in the solutions when K_2 and K_3 are reduced to $K_2 = K_3 = 0.01$ and $K_2 = K_3 = 0.001$, respectively. As mentioned previously, the initial guess for a new set of values of K_2 and K_3 is the solution with the previous values of K_2 and K_3.

With K_3 fixed at 0.001, if K_2 is increased (Figs. 1-14 and 1-15), the solutions will change, until $K_2 = 8$ for solution no. 1 and $K_2 = 7$ for solution no. 2, beyond which no solution is obtained. It is interesting to note that the two solutions move closer together with increasing K_2 and almost merge into each other before disappearing altogether. The result is analogous to lowering the $G(j\omega)$ curve or raising the $-1/N$ curve in the single-axis case.

It can be concluded that for $K_3 = 0.001$, K_2 should be less than 8 for a sustained oscillation to occur. In fact, with $K_2 = 8$ (solution 1) and $K_2 = 7$ (solution 2) the roots of the stability equations do not appear to have converged adequately, and well defined roots occur only for lower values of K_2. A maximum of 20 iterations are attempted before the numerical process is terminated.

If K_3 is kept fixed at 0.2 and K_2 is varied, the iterations in

Figs. 1-16 and 1-17 show that K_2 can be increased only up to $K_2 = 3$ for solutions to be obtained.

On the other hand, K_2 can be kept fixed and K_3 varied; these results are shown in Figs. 1-18 through 1-25. In Figs. 1-18 and 1-19 K_2 is fixed at 0.001, and the maximum value of K_3 for a solution to the stability equation is approximately $K_3 = 0.4$ for solution no. 1 and $K_2 = 0.3$ for solution no. 2. With $K_2 = 0.1$, 1.0 and 5.0, the maximum values of K_3 are 0.4 (solution 1 and 2), 0.3 (solution 1 and 2) and <0.1 (solution 1 and 2), respectively. These results are shown in Figs. 1-20 through 1-25.

Table 1-1 summarizes all the results. In the table, the solutions for $K_2 = K_3 = 0$ are the solutions with the single-axis LST system. It is seen that when K_2 and K_3 are very small, the solutions of the two-axis system approach those of the single-axis system.

The data in Table 1-1 can be used to plot the region in the $K_2 - K_3$ plane in which sustained oscillation can exist. Figure 1-26 shows this region. For values of K_2 and K_3 outside the crosshatched area, no sustained oscillations should exist.

The motion of the roots with variable K_2 and K_3 are plotted in Fig. 1-27, using the data in Table 1-1. The points marked $K_2 = 0$, $K_3 = 0$, represent the single-axis LST system solutions. The solid closed regions represent the regions of root movements where reliable convergence was obtained. The dotted lines include the regions where reliable convergence of the iterative scheme was not obtained with 20 iterations, and, which can be considered as a boundary area between existence and non-existence of solutions. Note that in Fig. 1-27 the two solutions are farthest apart when $K_2 = K_3 = 0$. Figure 1-27 also shows how the two

solutions tend to merge just before the solutions disappear.

Table 1-1

K_2	K_3	ROOT #1		ROOT #2	
		FREQUENCY	AMPLITUDE	FREQUENCY	AMPLITUDE
0.1	0.1	4.1	5.07×10^{-6}	1.8	5.99×10^{-7}
0.01	0.01	4.26	5.52×10^{-6}	1.76	4.58×10^{-7}
0.001	0.001	4.27	5.56×10^{-6}	1.75	4.45×10^{-7}
0.0	0.0	4.27	5.57×10^{-6}	1.75	4.45×10^{-7}
0.01	0.001	4.27	5.57×10^{-6}	1.75	4.46×10^{-7}
0.1	0.001	4.28	5.55×10^{-6}	1.79	4.59×10^{-7}
1.0	0.001	4.31	5.24×10^{-6}	2.11	5.79×10^{-7}
2.0	0.001	4.34	4.91×10^{-6}	2.43	7.17×10^{-7}
3.0	0.001	4.36	4.56×10^{-6}	2.73	8.69×10^{-7}
4.0	0.001	4.38	4.20×10^{-6}	3.01	1.04×10^{-6}
5.0	0.001	4.37	3.81×10^{-6}	3.29	1.25×10^{-6}
6.0	0.001	4.34	3.35×10^{-6}	3.58	1.53×10^{-6}
7.0	0.001	4.19	2.6×10^{-6}	3.97**	2.06×10^{-6}**
8.0	0.001	3.95**	1.50×10^{-6}**	-	-
1.0	0.001	4.31	5.24×10^{-6}	2.11	5.79×10^{-7}
1.0	0.01	4.29	5.19×10^{-6}	2.23	5.93×10^{-7}
1.0	0.1	4.13	4.74×10^{-6}	2.12	7.71×10^{-7}
1.0	0.2	3.91	4.18×10^{-6}	2.39	1.03×10^{-6}
1.0	0.3	3.62	3.47×10^{-6}	2.62	1.43×10^{-6}
1.0	0.4	-	-	3.2**	2.57×10^{-6}**
5.0	0.001	4.37	3.81×10^{-6}	3.29	1.25×10^{6}
5.0	0.01	4.34	3.72×10^{-6}	3.31	1.30×10^{6}
5.0	0.1	5.46**	7.13×10^{-6}**	3.75**	2.25×10^{6}**
1.0	0.2	3.91	4.17×10^{-6}	2.39	1.04×10^{-6}
2.0	0.2	3.85	3.64×10^{-6}	2.79	1.37×10^{-6}
3.0	0.2	3.69	2.86×10^{-6}	3.28	1.95×10^{-6}
0.001	0.001	4.27	5.57×10^{-6}	1.75	4.45×10^{-7}
0.001	0.01	4.26	5.54×10^{-6}	1.76	4.56×10^{-7}

Table 1-1 (cont'd)

K_2	K_3	ROOT #1		ROOT #2	
		FREQUENCY	AMPLITUDE	FREQUENCY	AMPLITUDE
0.001	0.1	4.11	5.12×10^{-6}	1.84	5.79×10^{-7}
0.001	0.2	3.92	4.63×10^{-6}	1.95	7.52×10^{-7}
0.001	0.3	3.7	4.08×10^{-6}	2.1	9.82×10^{-7}
0.001	0.4	3.41	3.42×10^{-6}	-	-
0.001	0.5	2.94**	2.39×10^{-6}	-	-
0.1	0.1	4.1	5.08×10^{-6}	1.88	5.98×10^{-7}
0.1	0.2	3.92	4.59×10^{-6}	2.0	7.79×10^{-7}
0.1	0.3	3.7	4.03×10^{-6}	2.15	1.02×10^{-6}
0.1	0.4	3.40	3.34×10^{-6}	2.38	1.39×10^{-6}
0.1	0.5	-	-	2.82**	2.22×10^{-6}**

** Solution did not converge in 20 iterations and is not considered reliable.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-01     K3= 1.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.5000000000D 01     0.5000000000D-05
      1     0.4733762972D 01     0.5433071349D-05
      2     0.4526562584D 01     0.5454333473D-05
      3     0.4375323004D 01     0.5360015536D-05
      4     0.4270975962D 01     0.5259907533D-05
      5     0.4203232573D 01     0.5183187332D-05
      6     0.4161323855D 01     0.5132000856D-05
      7     0.4136226451D 01     0.5100190831D-05
      8     0.4121313005D 01     0.5080665473D-05
      9     0.4111573450D 01     0.5066187720D-05
     10     0.4125227288D 01     0.5118030600D-05
     11     0.4115488437D 01     0.5099437398D-05
     12     0.4104483976D 01     0.5075156770D-05
     13     0.4108951055D 01     0.5075090383D-05
     14     0.4108776583D 01     0.5074449931D-05

ND= 1   FREQUENCY= 4.10860D 00RAD/SEC   AMPLITUDE= 5.07397D-06   NIT= 14
```



```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-02     K3= 1.00000D-02

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.4108597203D 01     0.5073966584D-05
      1     0.4168820145D 01     0.5248288860D-05
      2     0.4205531785D 01     0.5364353532D-05
      3     0.4226335066D 01     0.5432259787D-05
      4     0.4237936472D 01     0.5470663581D-05
      5     0.4244508200D 01     0.5492764659D-05
      6     0.4246721288D 01     0.5477545295D-05
      7     0.4253298522D 01     0.5511028750D-05
      8     0.4253818862D 01     0.5517540846D-05
      9     0.4253573182D 01     0.5522607930D-05
     10     0.4262696521D 01     0.5520588880D-05
     11     0.4255858006D 01     0.5512477452D-05
     12     0.4255246471D 01     0.5516839666D-05

ND= 1   FREQUENCY= 4.25522D 00RAD/SEC   AMPLITUDE= 5.52114D-06   NIT= 12

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-03     K3= 1.00000D-03

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.4255217824D 01     0.5521142485D-05
      1     0.4260676424D 01     0.5542795818D-05
      2     0.4256534816D 01     0.5527396968D-05
      3     0.4252613070D 01     0.5420675622D-05
      4     0.4261846964D 01     0.5496517630D-05
      5     0.4266693459D 01     0.5536135509D-05
      6     0.4269530544D 01     0.5557835647D-05
      7     0.4269042818D 01     0.5561545739D-05

ND= 1   FREQUENCY= 4.26902D 00RAD/SEC   AMPLITUDE= 5.56645D-06   NIT= 7
```

Figure 1-12.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.2000000000D 01     0.1000000000D-05
        1     0.1849233024D 01     0.6242848602D-06
        2     0.1852950575D 01     0.5882981170D-06
        3     0.1867139237D 01     0.5930626214D-06
        4     0.1874829545D 01     0.5962272806D-06
        5     0.1878890231D 01     0.5979138586D-06
        6     0.1880129011D 01     0.5979328084D-06
        7     0.1880201346D 01     0.5986153910D-06
        8     0.1880334350D 01     0.5986705996D-06

ND= 1   FREQUENCY= 1.88042D 00RAD/SEC   AMPLITUDE= 5.98812D-07   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-02      K3= 1.00000D-02

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1880421807D 01     0.5988118404D-06
        1     0.1816575499D 01     0.5204743608D-06
        2     0.1787791116D 01     0.4874090332D-06
        3     0.1774071480D 01     0.4720024797D-06
        4     0.1767310255D 01     0.4645333492D-06
        5     0.1763726929D 01     0.4605878423D-06
        6     0.1758874217D 01     0.4525793409D-06
        7     0.1758812030D 01     0.4613877605D-06
        8     0.1758882166D 01     0.4583095382D-06
        9     0.1759606827D 01     0.4577612065D-06
       10     0.1759617287D 01     0.4581187707D-06

ND= 1   FREQUENCY= 1.75963D 00RAD/SEC   AMPLITUDE= 4.58129D-07   NIT= 10


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1759634012D 01     0.4581286762D-06
        1     0.1754793059D 01     0.4515838346D-06
        2     0.1752096824D 01     0.4481194314D-06
        3     0.1743716093D 01     0.4383366512D-06
        4     0.1744785219D 01     0.4403176104D-06
        5     0.1746624895D 01     0.4423270886D-06
        6     0.1747172576D 01     0.4436675832D-06
        7     0.1747595835D 01     0.4442578928D-06
        8     0.1747828656D 01     0.4446765861D-06

ND= 1   FREQUENCY= 1.74799D 00RAD/SEC   AMPLITUDE= 4.44921D-07   NIT=  8
```


Figure 1-13.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-02      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4269023768D 01     0.5566448250D-05
        1     0.4260430902D 01     0.5563841178D-05
        2     0.4267484262D 01     0.5564859360D-05
        3     0.4272363751D 01     0.5564759926D-05
        4     0.4272512787D 01     0.5571703289D-05
        5     0.4272700920D 01     0.5571487431D-05

ND= 1   FREQUENCY= 4.27286D 00RAD/SEC   AMPLITUDE= 5.57162D-06   NIT=  5


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4272860404D 01     0.5571617068D-05
        1     0.4277637363D 01     0.5561946869D-05
        2     0.4275891190D 01     0.5550585436D-05
        3     0.4275463586D 01     0.5545271994D-05

ND= 1   FREQUENCY= 4.27574D 00RAD/SEC   AMPLITUDE= 5.54293D-06   NIT=  3


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4275736610D 01     0.5542934874D-05
        1     0.4335235297D 01     0.5481663112D-05
        2     0.4324421026D 01     0.5365104085D-05
        3     0.4316383079D 01     0.5303160811D-05
        4     0.4312372981D 01     0.5272793022D-05
        5     0.4310288669D 01     0.5257432430D-05
        6     0.4309114710D 01     0.5249302855D-05
        7     0.4307622250D 01     0.5243785475D-05
        8     0.4302486615D 01     0.5236904245D-05
        9     0.4307060453D 01     0.5237003046D-05
       10     0.4306932848D 01     0.5236915323D-05

ND= 1   FREQUENCY= 4.30676D 00RAD/SEC   AMPLITUDE= 5.23683D-06   NIT= 10
```

Figure 1-14a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4300000000D 01     0.5200000000D-05
        1     0.4305281886D 01     0.5228303901D-05
        2     0.4305283850D 01     0.5228453439D-05
        3     0.4308744870D 01     0.5245748728D-05
        4     0.4308634353D 01     0.5243241015D-05

ND= 1   FREQUENCY= 4.30865D 00RAD/SEC   AMPLITUDE= 5.24185D-06   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 2.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4308645193D 01     0.5241850229D-05
        1     0.4375960005D 01     0.5185863190D-05
        2     0.4360033500D 01     0.5047381412D-05
        3     0.4349924772D 01     0.4976486582D-05
        4     0.4344905380D 01     0.4941913249D-05
        5     0.4342313852D 01     0.4924475958D-05
        6     0.4340863107D 01     0.4915247605D-05
        7     0.4339884306D 01     0.4909947550D-05
        8     0.4339062983D 01     0.4906586223D-05

ND= 1   FREQUENCY= 4.33810D 00RAD/SEC   AMPLITUDE= 4.90415D-06   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 3.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4338100273D 01     0.4904152108D-05
        1     0.4407947330D 01     0.4861009128D-05
        2     0.4387383490D 01     0.4711098352D-05
        3     0.4375506949D 01     0.4636301815D-05
        4     0.4369644072D 01     0.4599948927D-05
        5     0.4366618831D 01     0.4581578394D-05
        6     0.4364918225D 01     0.4571778521D-05
        7     0.4363815507D 01     0.4566133105D-05
        8     0.4362939491D 01     0.4562548417D-05

ND= 1   FREQUENCY= 4.36199D 00RAD/SEC   AMPLITUDE= 4.55997D-06   NIT=  8
```


Figure 1-14b.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 4.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43619881750 01     0.4559965684D-05
        1     0.44324403670 01     0.4525930616D-05
        2     0.44056315540 01     0.4360942028D-05
        3     0.43912620160 01     0.4280822026D-05
        4     0.43842438520 01     0.4242108964D-05
        5     0.43806287320 01     0.4222494599D-05
        6     0.43785908300 01     0.4211923921D-05
        7     0.43773946740 01     0.4205893989D-05
        8     0.43766507050 01     0.4202218261D-05

ND= 1   FREQUENCY= 4.37615D 00RAD/SEC   AMPLITUDE= 4.19983D-06   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 5.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43761486130 01     0.4199830027D-05
        1     0.44462686900 01     0.4172529128D-05
        2     0.44107233930 01     0.3986239376D-05
        3     0.43923162960 01     0.3897312851D-05
        4     0.43834979140 01     0.3854826622D-05
        5     0.43789660240 01     0.3833214180D-05
        6     0.43763990410 01     0.3821392659D-05
        7     0.43748983080 01     0.3814536904D-05
        8     0.43740363500 01     0.3810350253D-05
        9     0.43735670320 01     0.3807689161D-05

ND= 1   FREQUENCY= 4.37333D 00RAD/SEC   AMPLITUDE= 3.80595D-06   NIT=  9


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 6.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43733253690 01     0.3805950594D-05
        1     0.44436880830 01     0.3786225692D-05
        2     0.43937160140 01     0.3563944711D-05
        3     0.43669065680 01     0.3456015384D-05
        4     0.43545475270 01     0.3405764070D-05
        5     0.43481627460 01     0.3379916365D-05
        6     0.43444856440 01     0.3365382272D-05
        7     0.43422667430 01     0.3356649144D-05
        8     0.43411479830 01     0.3351365017D-05
        9     0.43408331140 01     0.3348264128D-05

ND= 1   FREQUENCY= 4.34081D 00RAD/SEC   AMPLITUDE= 2.34647D-06   NIT=  9
```


Figure 1-14c.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 7.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43408123810 01     0.33464733730-05
        1     0.44086450490 01     0.33257897580-05
        2     0.43250391460 01     0.30185972570-05
        3     0.42627017970 01     0.28282940470-05
        4     0.42331822050 01     0.27389890860-05
        5     0.42165306160 01     0.26889250650-05
        6     0.42049353060 01     0.26555451940-05
        7     0.41978962930 01     0.26341001500-05
        8     0.41887513270 01     0.26126197740-05
        9     0.41897240440 01     0.26064416570-05
       10     0.41912115360 01     0.26044464500-05

ND= 1   FREQUENCY= 4.19137D 00RAD/SEC   AMPLITUDE= 2.60430D-06   NIT= 10


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 8.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.41913665860 01     0.26043009650-05
        1     0.43208305670 01     0.27554499200-05
        2     0.38856546740 01     0.15372275460-05
        3     0.39926889640 01     0.17770982030-05
        4     0.41012112540 01     0.20181976690-05
        5     0.40490681570 01     0.21337527150-05
        6     0.42186508680 01     0.24428080160-05
        7     0.53285410390 01     0.51215428080-05
        8     0.49323740970 01     0.40903961610-05
        9     0.45807429900 01     0.28987634790-05
       10     0.42726831170 01     0.23389309590-05
       11     0.47260127000 01     0.35272360130-05
       12     0.46424620320 01     0.22062142870-05
       13     0.41023851610 01     0.17126075280-05
       14     0.45110009400 01     0.28076571010-05
       15     0.41850494310 01     0.21439952400-05
       16     0.43379215410 01     0.25663857830-05
       17     0.45847192910 01     0.31723960640-05
       18     0.44103131360 01     0.25954176160-05
       19     0.39015337200 01     0.14570902020-05
       20     0.39513553890 01     0.15000364790-05

ND= 1   FREQUENCY= 3.95136D 00RAD/SEC   AMPLITUDE= 1.50004D-06   NIT= 20
```

Figure 1-14d.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-02      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1747994849D 01     0.4449210692D-06
        1     0.1750970353D 01     0.4459628356D-06
        2     0.1751578924D 01     0.4458610524D-06
        3     0.1751663855D 01     0.4463090840D-06
        4     0.1751762687D 01     0.4463992173D-06

ND= 1   FREQUENCY= 1.75183D 00RAD/SEC   AMPLITUDE= 4.46504D-07   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1751834176D 01     0.4465041105D-06
        1     0.1769294195D 01     0.4510101543D-06
        2     0.1778904699D 01     0.4546263062D-06
        3     0.1784009536D 01     0.4567644431D-06
        4     0.1786666712D 01     0.4577447044D-06
        5     0.1786879223D 01     0.4579707749D-06

ND= 1   FREQUENCY= 1.78701D 00RAD/SEC   AMPLITUDE= 4.58193D-07   NIT=  5


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1787010012D 01     0.4581926081D-06
        1     0.1866463182D 01     0.3314511173D-06
        2     0.1953999120D 01     0.4148348970D-06
        3     0.2021290853D 01     0.4825286553D-06
        4     0.2064528709D 01     0.5264398004D-06
        5     0.2089101053D 01     0.5514356079D-06
        6     0.2101978469D 01     0.5644300305D-06
        7     0.2108266521D 01     0.5706690202D-06
        8     0.2110689098D 01     0.5735103622D-06
        9     0.2111477387D 01     0.5755678257D-06
       10     0.2112135359D 01     0.5764854872D-06
       11     0.2112491813D 01     0.5772192262D-06
       12     0.2112768819D 01     0.5776432494D-06

ND= 1   FREQUENCY= 2.11296D 00RAD/SEC   AMPLITUDE= 5.77976D-07   NIT= 12
```

Figure 1-15a.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2100000000D 01	0.5800000000D-06
1	0.2105819298D 01	0.5764408218D-06
2	0.2110683445D 01	0.5777939437D-06
3	0.2113400637D 01	0.5784885398D-06
4	0.2113458576D 01	0.5787551958D-06

ND= 1 FREQUENCY= 2.11352D 00RAD/SEC AMPLITUDE= 5.78829D-07 NIT= 4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 2.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2113522368D 01	0.5788294272D-06
1	0.2198783766D 01	0.4745734256D-06
2	0.2286273095D 01	0.5541505311D-06
3	0.2351691717D 01	0.6233706268D-06
4	0.2392340682D 01	0.6677152962D-06
5	0.2414543718D 01	0.6919494671D-06
6	0.2425574373D 01	0.7037403848D-06
7	0.2430351540D 01	0.7088785615D-06
8	0.2431880854D 01	0.7120244100D-06
9	0.2432839304D 01	0.7138996007D-06
10	0.2433482653D 01	0.7151060090D-06
11	0.2433935486D 01	0.7159386678D-06
12	0.2434270538D 01	0.7165388026D-06

ND= 1 FREQUENCY= 2.43453D 00RAD/SEC AMPLITUDE= 7.16991D-07 NIT= 12

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 3.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2434527302D 01	0.7169908360D-06
1	0.2530703725D 01	0.6753917942D-06
2	0.2615049668D 01	0.7473016901D-06
3	0.2671818957D 01	0.8057580249D-06
4	0.2703849298D 01	0.8395671944D-06
5	0.2719819820D 01	0.8557534471D-06
6	0.2726656058D 01	0.8617656173D-06
7	0.2728173461D 01	0.8639006049D-06
8	0.2728919117D 01	0.8658976396D-06
9	0.2729603251D 01	0.8669499962D-06
10	0.2730045451D 01	0.8679340734D-06
11	0.2730421824D 01	0.8685866208D-06

ND= 1 FREQUENCY= 2.73071D 00RAD/SEC AMPLITUDE= 8.69150D-07 NIT= 11

Figure 1-15b.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.380000 07 TO= 1.000000D-01 JV= 1.000000D 05 JG= 2.100000D 00
KP= 2.160000 02 KI= 9.700000D 03 H= 6.000000D 02 K0= 5.75835D 03
K1= 1.371020 03 K2= 4.000000 00 K3= 1.000000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.27307112410 01	0.8691498890D-06
1	0.28327612720 01	C.8690562992D-06
2	0.29140723690 01	0.9404417447D-06
3	0.29652075090 01	0.9931230333D-06
4	0.29927995950 01	0.1021781452D-05
5	0.30063065160 01	0.1034869396D-05
6	0.30111433820 01	0.1038001938D-05
7	0.30116053060 01	0.1040285461D-05
8	0.30122946140 01	0.1040916602D-05
9	0.30126669600 01	0.1041986665D-05
10	0.30130411490 01	0.1042589602D-05

ND= 1 FREQUENCY= 3.01333D 00RAD/SEC AMPLITUDE= 1.04321D-06 NIT= 10

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.380000 07 TO= 1.000000D-01 JV= 1.000000D 05 JG= 2.100000D 00
KP= 2.160000 02 KI= 9.700000D 03 H= 6.000000D 02 K0= 5.75835D 03
K1= 1.371020 03 K2= 5.000000 00 K3= 1.000000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.30133297290 01	0.1043207423D-05
1	0.31158855480 01	0.1065992225D-05
2	C.31957166540 01	0.1142568839D-05
3	0.32459575300 01	0.1198719877D-05
4	0.32735744710 01	0.1230191890D-05
5	0.32869829930 01	0.1244194422D-05
6	0.32899643840 01	0.1245681245D-05
7	0.32904455500 01	0.1248731532D-05
8	0.32913330090 01	0.1249376945D-05
9	0.32917659820 01	0.1250826172D-05
10	0.32922447670 01	0.1251583502D-05

ND= 1 FREQUENCY= 3.29261D 00RAD/SEC AMPLITUDE= 1.25244D-06 NIT= 10

Figure 1-15c.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00

KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03

K1= 1.37102D 03 K2= 6.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.32926089030 01	0.12524417490-05
1	0.33931182040 01	0.12880153440-05
2	0.34766480040 01	0.13821226540-05
3	0.35324191450 01	0.14555935280-05
4	0.35645857580 01	0.14992048020-05
5	0.35782924460 01	0.15150643390-05
6	0.35799595170 01	0.15178912160-05
7	0.35810681380 01	0.15209709710-05
8	0.35821534120 01	0.15230940250-05
9	0.35829991130 01	0.15251788380-05
10	0.35837765440 01	0.15268278690-05
11	0.35844360030 01	0.15283495600-05

ND= 1 FREQUENCY= 3.58503D 00RAD/SEC AMPLITUDE= 1.52965D-06 NIT= 11

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00

KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03

K1= 1.37102D 03 K2= 7.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.35850294550 01	0.15296501160-05
1	0.36835499180 01	0.15758265830-05
2	0.37823023170 01	0.16157825650-05
3	0.38603849890 01	0.16458471720-05
4	0.39100843630 01	0.16930617649-05
5	0.39215573620 01	0.16473093000-05
6	0.39279550100 01	0.16590148480-05
7	0.39317294400 01	0.16900152340-05
8	0.39497523190 01	0.20140214810-05
9	0.39509208140 01	0.20242424210-05
10	0.39547837670 01	0.20274338040-05
11	0.39560248320 01	0.20330211970-05
12	0.39578890570 01	0.20357573530-05
13	0.39591420550 01	0.20396195410-05
14	0.39605362450 01	0.20424659060-05
15	0.39617190700 01	0.20455586640-05
16	0.39628987570 01	0.20482462340-05
17	0.39639796620 01	0.20509056350-05
18	0.39650208010 01	0.20533612810-05
19	0.39660004090 01	0.20557231180-05
20	0.39669361610 01	0.20579522940-05

ND= 1 FREQUENCY= 3.96694D 00RAD/SEC AMPLITUDE= 2.05795D-06 NIT= 20

Figure 1-15d.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3900000000D 01     0.4200000000D-05
        1     0.3911764448D 01     0.4198154369D-05
        2     0.3910445465D 01     0.4185610296D-05
        3     0.3907288862D 01     0.4175505017D-05
        4     0.3909715946D 01     0.4174222859D-05

ND= 1   FREQUENCY= 3.91004D 00RAD/SEC   AMPLITUDE= 4.17378D-06   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTAON OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 2.00000D 00      K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3910038835D 01     0.4173781422D-05
        1     0.3944858612D 01     0.4034291927D-05
        2     0.3902493997D 01     0.3834529223D-05
        3     0.3880015397D 01     0.3738445839D-05
        4     0.3869325567D 01     0.3692951635D-05
        5     0.3863963301D 01     0.3670144295D-05
        6     0.3861018723D 01     0.3657865665D-05
        7     0.3859279764D 01     0.3650781946D-05
        8     0.3858194339D 01     0.3646410253D-05
        9     0.3857484154D 01     0.3643544224D-05

ND= 1   FREQUENCY= 3.85700D 00RAD/SEC   AMPLITUDE= 3.64157D-06   NIT=  9


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 3.00000D 00      K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3856997882D 01     0.3641569608D-05
        1     0.3868151877D 01     0.3446356451D-05
        2     0.3795140139D 01     0.3178207222D-05
        3     0.3744743850D 01     0.3023885812D-05
        4     0.3720285047D 01     0.2950694083D-05
        5     0.3708526518D 01     0.2914399703D-05
        6     0.3700756579D 01     0.2891913462D-05
        7     0.3696893587D 01     0.2879208811D-05
        8     0.3689410851D 01     0.2864645745D-05
        9     0.3692449829D 01     0.2862905116D-05

ND= 1   FREQUENCY= 3.692(4D 00RAD/SEC   AMPLITUDE= 2.86133D-06   NIT=  9
```

Figure 1-16.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D 00     K3= 2.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.2400000000D 01     0.1000000000D-05
      1     0.2409479441D 01     0.1046712564D-05
      2     0.2399290724D 01     0.1038804169D-05
      3     0.2394536278D 01     0.1035725356D-05
      4     0.2390864967D 01     0.1031368943D-05
      5     0.2395076850D 01     0.1046663017D-05
      6     0.2398404185D 01     0.1051367019D-05
      7     02400539326D 01     0.1042485398D-05
      8     0.2392532346D 01     0.1031818835D-05
      9     0.2392904613D 01     0.1037398336D-05
     10     0.2393401118D 01     0.1040674411D-05
     11     0.2393603834D 01     0.1040720365D-05

ND= 1   FREQUENCY= 2.39389D 00RAD/SEC   AMPLITUDE= 1.04135D-06   NIT= 11


LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 2.00000D 00     K3= 2.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.2393886185D 01     0.1041351791D-05
      1     0."545583278D 01     0.1115190202D-05
      2     0.2658949567D 01     0.1222449668D-05
      3     0.2730043552D 01     0.1298874812D-05
      4     0.2768736905D 01     0.1341388924D-05
      5     0.2787645362D 01     0.1361207870D-05
      6     0.2794552219D 01     0.1366690838D-05
      7     0.2795388620D 01     0.1368716008D-05
      8     0.2796186906D 01     0.1370021105D-05

ND= 1   FREQUENCY= 2.79679D 00RAD/SEC   AMPLITUDE= 1.37127D-06   NIT=  8


LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 3.00000D 00     K3= 2.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.2796785954D 01     0.1371271994D-05
      1     0.2957809532D 01     0.1502914789D-05
      2     0.3082893020D 01     0.1654250751D-05
      3     0.3172400981D 01     0.1779866320D-05
      4     0.3229810908D 01     0.1865488514D-05
      5     0.3255267109D 01     0.1900803739D-05
      6     0.3260443833D 01     0.1908847559D-05
      7     0.3264018726D 01     0.1916707271D-05
      8     0.3266798778D 01     0.1923509780D-05
      9     0.3269286411D 01     0.1928721353D-05
     10     0.3271371276D 01     0.1933362076D-05
     11     0.3273230827D 01     0.1937306384D-05
     12     0.3274855980D 01     0.1940826760D-05
     13     0.3276210806D 01     0.1948728198D-05
     15     0.3278302721D 01     0.1949234815D-05
     16     0.3279876254D 01     0.1951518427D-05
     17     0.3280857769D 01     0.1953599601D-05
     18     0.3281758233D 01     0.1955508065D-05

ND= 1   FREQUENCY= 3.28259D 00RAD/SEC   AMPLITUDE= 1.95726D-06   NIT= 18
```

Figure 1-17.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03     K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43000000000D 01     0.5500000000D-05
        1     0.4271086058D 01      0.5508161796D-05
        2     0.4265786694D 01      0.5527424953D-05
        3     0.4268750642D 01      0.5552197589D-05
        4     0.4290753071D 01      0.5645724078D-05
        5     0.4278015463D 01      0.5604179382D-05
        6     0.4271073196D 01      0.5582480418D-05
        7     0.4272476170D 01      0.5579216017D-05

ND= 1   FREQUENCY= 4.27381D 00RAD/SEC   AMPLITUDE= 5.57884D-06   NIT=  7


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03     K3= 1.00000D-02

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4273811433D 01      0.5578838989D-05
        1     0.4265165280D 01      0.5561562337D-05
        2     0.4256891908D 01      0.5539854734D-05
        3     0.4258571915D 01      0.5537666565D-05

ND= 1   FREQUENCY= 4.25947D 00RAD/SEC   AMPLITUDE= 5.53740D-06   NIT=  3


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03     K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4259473371D 01      0.5537397133D-05
        1     0.4196212759D 01      0.5385323809D-05
        2     0.4154989128D 01      0.5265320509D-05
        3     0.4130148775D 01      0.5190949316D-05
        4     0.4115216330D 01      0.5146566295D-05
        5     0.4107031778D 01      0.5123226637D-05
        6     0.4109286422D 01      .5120752653D-05

ND= 1   FREQUENCY= 4.11028D 00RAD/SEC   AMPLITUDE= 5.12006D-06   NIT=  6
```


Figure 1-18a.

```
LST SYSTEM-NUMERIAAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-03     K= 2.00000D-01

  ITERATION    FREQUENCY            AMPLITUDE
      0    0.4110284514D 01    0.5120057257D-05
      1    0.4032019881D 01    0.4940948570D-05
      2    0.3980474639D 01    0.4799138365D-05
      3    0.3948897805D 01    0.4710444853D-05
      4    0.3929525533D 01    0.4656795911D-05
      5    0.3920240815D 01    0.4631418674D-05
      6    0.3922604706D 01    0.4628602863D-05

ND= 1   FREQUENCY= 3.92377D 00RAD/SEC   AMPLITUDE= 4.62766D-06   NIT=  6
```



```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.0000D-03     K3= 3.00000D-01

  ITERATION    FREQUENCY            AMPLITUD

      0    0.3923766777D 01    0.4627662693D-05
      1    0.3832689293D 01    0.4430973619D-05
      2    0.3770977701D 01    0.4271935832D-05
      3    0.3731825309D 01    0.4170216481D-05
      4    0.3706733818D 01    0.4106969040D-05
      5    0.3702042847D 01    0.4090408277D-05
      6    0.3701950079D 01    0.4084753797D-05
      7    0.3704511777D 01    0.4084614961D-05

ND= 1   FREQUENCY= 3.70495D 00RAD/SEC   AMPLITUDE= 4.08573D-06   NIT=  7

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-03     K3= 4.00000D-01

  ITERATION    FREQUENCY            AMPLITUDE
      0    0.3704948126D 01    0.4085726824D-05
      1    0.3590600302D 01    0.3852326005D-05
      2    0.3508515778D 01    0.3657206541D-05
      3    0.3451606441D 01    0.3523805145D-05
      4    0.3412420645D 01    0.3436756348D-05
      5    0.3416493548D 01    0.3433169185D-05
      6    0.3408329008D 01    0.3418746939D-05
      7    0.3412284461D 01    0.3418436857D-05
      8    0.3412294387D 01    0.3417129272D-05

ND= 1   FREQUENCY= 3.41234D 00RAD/SEC   AMPLITUDE= 3.41641D-06   NIT=  8
```

Figure 1-18b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03      K3= 5.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3412341331D 01     0.3416405203D-05
        1     0.3241064820D 01     0.3083542987D-05
        2     0.3077393747D 01     0.2738920401D-05
        3     0.2804995297D 01     0.2210792560D-05
        4     0.2920590092D 01     0.2394592414D-05
        5     0.2947896527D 01     0.2746799724D-05
        6     0.3223599934D 01     0.3158399913D-05
        7     0.3039734314D 01     0.2693238377D-05
        8     0.2821487226D 01     0.2247034327D-05
        9     0.3001552640D 01     0.2542816655D-05
       10     0.3097700472D 01     0.2720433834D-05
       11     0.2749506483D 01     0.1990339067D-05
       12     0.2948088164D 01     0.2408982921D-05
       13     0.3295723942D 01     0.3130139548D-05
       14     0.3159659222D 01     0.3069275282D-05
       15     0.3022611977D 01     0.2679898335D-05
       16     0.n858930557D 01     0.2324232956D-05
       17     0.468415503D 01     0.3383591544D-05
       18     0.3320392030D 01     0.3057606152D-05
       19     0.3141388572D 01     0.2765634620D-05
       20     0.2941220353D 01     0.2394984045D-05

ND= 1   FREQUENCY= 2.94122D 00RAD/SEC   AMPLITUDE= 2.39H98D-06   NIT= 20


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03      K3= 6.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.2941220353D 01     0.2394984045D-05
        1     0.3168814268D 01     0.3627195910D-05
        2     0.2703307611D 01     0.2052990628D-05
        3     0.3160507042D 01     0.2863909229D-05
        4     0.2695124944D 01     0.2013043232D-05
        5     0.3115544834D 01     0.2763671945D-05
        6     0.2633809736D 01     0.1937538081D-05
        7     0.2999865321D 01     0.2567990764D-05
        8     0.1973349554D 01     0.6483543094D-06
        9     0.2053560184D 01     09097663048D-06
       10     0.2256811360D 01     0.1229528999D-05
       11     0.2485081417D 01     0.1592105676D-05
       12     0.2952122537D 01     0.2199870999D-05
       13    -0.5763678769D 01    -0.1319216559D-04
  1    -5.76368D 00    -1.31922D-05
       14     0.9999999996D 00     0.9999999996D 00
       15     0.1000000000D 01     0.1000000000D 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT= 15
```

Figure 1-18c.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.17000000000D 01     0.45000000000D-06
        1     0.17196918940 01      0.43777776860-06
        2     0.17338172100 01      0.44017307660-06
        3     0.17416563900 01      0.44260183220-06
        4     0.17458916310 01      0.44416354490-06
        5     0.17480655370 01      0.44478128240-06
        6     0.17481415430 01      0.44517269960-06

ND= 1   FREQUENCY= 1.74823D 00RAD/SEC   AMPLITUDE= 4.45254D-07   NIT=  6


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03      K3= 1.00000D-02

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.17482272800 01      0.44525367750-06
        1     0.17521267780 01      0.45027962500-06
        2     0.17536501900 01      0.45297748390-06
        3     0.17544211610 01      0.45443790070-06
        4     0.17548888590 01      0.45521388440-06
        5     0.17551720260 01      0.45568452660-06
        6     0.17553626340 01      0.45597799570-06

ND= 1   FREQUENCY= 1.75550D 00RAD/SEC   AMPLITUDE= 4.56178D-07   NIT=  6


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03      K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.17554971910 01      0.45617849940-06
        1     0.17917793460 01      0.51016129250-06
        2     0.18137122150 01      0.54201842140-06
        3     0.18260818110 01      0.55958304870-06
        4     0.18326217210 01      0.56878979040-06
        5     0.18353480010 01      0.57388085600-06
        6     0.18364208200 01      0.57687422380-06
        7     0.18373880020 01      0.57801715240-06
        8     0.18378236260 01      0.57891074070-06
        9     0.18381747210 01      0.57936987230-06

ND= 1   FREQUENCY= 1.83840D 00RAD/SEC   AMPLITUDE= 5.79735D-07   NIT=  9
```

Figure 1-19a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03     K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
         0    0.1838400034D 01     0.5797354291D-06
         1    0.1886569754D 01     0.6511742156D-06
         2    0.1917483403D 01     0.6957220257D-06
         3    0.1936546542D 01     0.7211874011D-06
         4    0.1927945480D 01     0.7499418438D-06
         5    0.1938822886D 01     0.7468010454D-06
         6    0.1947515177D 01     0.7500348081D-06
         7    0.1952344239D 01     0.7521033896D-06
         8    0.1954077725D 01     0.7522564047D-06

ND= 1   FREQUENCY= 1.95414D 00RAD/SEC   AMPLITUDE= 7.52932D-07   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTIOF OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-03     K3= 3.00000D-01

   ITERATION     FREQUENCY              AMPITUDE
         0    0.1954136225D 01     0.7529324744D-06
         1    0.2015669259D 01     0.8437249890D-06
         2    0.2059812684D 01     0.9034212199D-06
         3    0.2063822449D 01     0.9620807453D-06
         4    0.2084573235D 01     0.9719225440D-06
         5    0.2096414115D 01     0.9788230989D-06
         6    0.2102542141D 01     0.9822800761D-06
         7    0.2104071397D 01     0.9823954219D-06

ND= 1   FREQUENCY= 2.10420D 00RAD/0EC   AMPLITUDE= 9.83224D-05   NIT=  7
```

Figure 1-19b.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4000000000D 01     0.5000000000D-05
        1     0.4069348489D 01     0.5060816759D-05
        2     0.4096002875D 01     0.5075146970D-05
        3     0.4105852505D 01     0.5077069171D-05
        4     0.4109947410D 01     0.5076838934D-05

ND= 1   FREQUENCY= 4.10996D 00RAD/SEC   AMPLITUDE= 5.07630D-06   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4109963752D 01     0.5076302789D-05
        1     0.4031709712D 01     0.4897281315D-05
        2     0.3980008704D 01     0.4755356981D-05
        3     0.3948237513D 01     0.4666438611D-05
        4     0.3928651596D 01     0.4612520582D-05
        5     0.3920000370D 01     0.4588512642D-05
        6     0.3922292760D 01     0.4585636143D-05

ND= 1   FREQUENCY= 3.92343D 00RAD/SEC   AMPLITUDE= 4.58466D-06   NIT=  6


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 3.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3923434006D 01     0.4584662591D-05
        1     0.3830573188D 01     0.4383986105D-05
        2     0.3767396170D 01     0.4221711154D-05
        3     0.3727073921D 01     0.4117495178D-05
        4     0.3701052335D 01     0.4052404157D-05
        5     0.3697334293D 01     0.4037389204D-05
        6     0.3696451845D 01     0.4030788857D-05
        7     0.3700278587D 01     0.4031793320D-05
        8     0.3700684880D 01     0.4034395196D-05

ND= 1   FREQUENCY= 3.70157D 00RAD/SEC   AMPLITUDE= 4.03611D-06   NIT=  8
```

Figure 1-20a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 4.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3701567171D 01     0.4036108745D-05
        1     0.3582816527D 01     0.3793426426D-05
        2     0.3496558168D 01     0.3589545285D-05
        3     0.3435511973D 01     0.3447780717D-05
        4     0.3392930038D 01     0.3354372236D-05
        5     0.3398193057D 01     0.3352297996D-05
        6     0.3392147326D 01     0.3340631474D-05
        7     0.3395177657D 01     0.3339691379D-05

ND= 1   FREQUENCY= 3.39539D 00RAD/SEC   AMPLITUDE= 3.33865D-06   NIT=  7


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 5.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3395385263D 01     0.3338654674D-05
        1     0.3209113520D 01     0.2976214976D-05
        2     0.3008614935D 01     0.2560313865D-05
        3     0.1185413321D 01    -0.8497375904D-06
  1     1.18541D 00   -8.49738D-07
        4     0.1000000098D 01     0.9999999038D 00
        5     0.1000000000D 01     0.1000000000D 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT=  5
```

Figure 1-20b.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1800000000D 01     0.6000000000D-06
        1     0.1829985273D 01     0.5796322434D-06
        2     0.1854650683D 01     0.5871916572D-06
        3     0.1868379408D 01     0.5930486354D-06
        4     0.1875561288D 01     0.5963240748D-06
        5     0.1879253743D 01     0.5978864735D-06
        6     0.1879958027D 01     0.5978609512D-06

ND= 1   FREQUENCY= 1.88008D 00RAD/SEC   AMPLITUDE= 5.98399D-07   NIT=  6


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1880077917D 01     0.5983992937D-06
        1     0.1929412767D 01     0.6723251343D-06
        2     0.1961241662D 01     0.7186786352D-06
        3     0.1981174432D 01     0.7452828246D-06
        4     0.1979638157D 01     0.7715334579D-06
        5     0.1990078720D 01     0.7747937864D-06
        6     0.1996130859D 01     0.7774268861D-06
        7     0.1999099796D 01     0.7783768185D-06
        8     0.1999207661D 01     0.7786887285D-06

ND= 1   FREQUENCY= 1.99932D 00RAD/SEC   AMPLITUDE= 7.78826D-07   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 3.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.1999323876D 01     0.7788262524D-06
        1     0.2062672759D 01     0.8734319104D-06
        2     0.2109122091D 01     0.9362249601D-06
        3     0.2115867393D 01     0.1007920039D-05
        4     0.2132853221D 01     0.1009006204D-05
        5     0.2145665678D 01     0.1016009078D-05
        6     0.2152649862D 01     0.1020051049D-05
        7     0.2155018071D 01     0.1020575884D-05
        8     0.2155123139D 01     0.1021438689D-05

ND= 1   FREQUENCY= 2.15534D 00RAD/SEC   AMPLITUDE= 1.02156D-06   NIT=  8
```

Figure 1-21a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 4.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.2155339261D 01     0.1021562747D-05
        1     0.2244225191D 01     0.1153988517D-05
        2     0.2085370885D 01     0.1261870534D-05
        3     0.2180247480D 01     0.1225745261D-05
        4     0.2271122614D 01     0.1290986319D-05
        5     0.2328959390D 01     0.1343089153D-05
        6     0.2361075624D 01     0.1373743172D-05
        7     0.2377184815D 01     0.1388691120D-05
        8     0.2382499388D 01     0.1392166003D-05
        9     0.2383050692D 01     0.1393893795D-05
       10     0.2383771714D 01     0.1394708494D-05

ND= 1   FREQUENCY= 2.38425D 00RAD/SEC   AMPLITUDE= 1.39567D-06   NIT= 10


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D-01      K3= 5.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.2384254008D 01     0.1395666760D-05
        1     0.2535609988D 01     0.1627192817D-05
        2     0.2630189741D 01     0.1909884571D-05
        3     0.2806366902D 01     0.2179832984D-05
        4     0.2979218718D 01     0.2469654213D-05
        5     0.2937010575D 01     0.2512698384D-05
        6     0.2589337751D 01     0.1773952983D-05
        7     0.2692365763D 01     0.1941448335D-05
        8     0.2790768007D 01     0.2105677939D-05
        9     0.2816230986D 01     0.2292847674D-05
       10     0.3214144945D 01     0.2985565780D-05
       11     0.3015994159D 01     0.2575097196D-05
       12     0.1819751304D 01     0.3275667481D-06
       13     0.1844100614D 01     0.5327149873D-06
       14     0.1997997856D 01     0.7845170266D-06
       15     0.2160355299D 01     0.1036885407D-05
       16     0.2316437477D 01     0.1281658930D-05
       17     0.2472457163D 01     0.1520150314D-05
       18     0.2503493328D 01     0.1807034130D-05
       19     0.2643365118D 01     0.1953910480D-05
       20     0.2819086976D 01     0.2217526558D-05

ND= 1   FREQUENCY= 2.81909D 00RAD/SEC   AMPLITUDE= 2.21753D-06   NIT= 20
```

Figure 1-21b.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43000000000D 01     0.52000000000D-05
        1     0.43052818860 01      0.5228303901D-05
        2     0.43052838500 01      0.5222453439D-05
        3     0.43087448700 01      0.5245748728D-05
        4     0.43086343530 01      0.5243241015D-05

ND= 1   FREQUENCY= 4.30865D 00RAD/SEC   AMPLITUDE= 5.24185D-06   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 1.00000D-02

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43086451930 01      0.5241850229D-05
        1     0.4298590514D 01      0.5220253018D-05
        2     0.42872949640 01      0.5194472604D-05
        3     0.42919135050 01      0.5194433798D-05
        4     0.4291762417D 01      0.5193867749D-05

ND= 1   FREQUENCY= 4.29161D 00RAD/SEC   AMPLITUDE= 5.19344D-06   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.42916075800 01      0.5193440699D-05
        1     0.4222646483D 01      0.5026200492D-05
        2     0.4177094770D 01      0.4895733868D-05
        3     0.41490950040 01      0.4814071540D-05
        4     0.4131739544D 01      0.4764607870D-05
        5     0.41262777780 01      0.4746822755D-05
        6     0.4128102003D 01      0.4743764169D-05

ND= 1   FREQUENCY= 4.12899D 00RAD/SEC   AMPLITUDE= 4.74269D-06   NIT=  6
```

Figure 1-22a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.4128986601D 01     0.4742691103D-05
        1     0.4038838818D 01     0.4534458733D-05
        2     0.3977078042D 01     0.4369916734D-05
        3     0.3938140899D 01     0.4264565926D-05
        4     0.3913052885D 01     0.4198877967D-05
        5     0.3909417445D 01     0.4188451086D-05
        6     0.3908686365D 01     0.4176817051D-05
        7     0.3910473948D 01     0.4175575370D-05

ND= 1   FREQUENCY= 3.91085D 00RAD/SEC   AMPLITUDE= 4.17559D-06   NIT=  7


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 3.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3910850693D 01     0.4175586358D-05
        1     0.3796023096D 01     0.3926734444D-05
        2     0.3712803035D 01     0.3720541479D-05
        3     0.3654173534D 01     0.3577665941D-05
        4     0.3613985577D 01     0.3485187376D-05
        5     0.3619420497D 01     0.3483188329D-05
        6     0.3611493035D 01     0.3468979466D-05
        7     0.3615626748D 01     0.3468727258D-05
        8     0.3615619356D 01     0.3467415619D-05

ND= 1   FREQUENCY= 3.61565D 00RAD/SEC   AMPLITUDE= 3.46667D-06   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 4.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.3615649056D 01     0.3466672496D-05
        1     0.3434297949D 01     0.3093089012D-05
        2     0.3240677649D 01     0.2669980182D-05
        3     0.1809721258D 01    -0.1572573193D-06
  1     1.80972D 00    -1.57257D-07
IHC900I EXECUTION TERMINATING DUE TO ERROR COUNT FOR ERROR NUMBER  261
```

Figure 1-22b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      KO= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D 00     K3= 1.00000D-03

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.21000000000D 01     0.58000000000D-06
      1     0.21058192980 01      0.57644082180-06
      2     0.21106834450 01      0.57779394370-06
      3     0.21134006370 01      0.57848853980-06
      4     0.21134585760 01      0.57875519580-06

ND= 1   FREQUENCY= 2.11352D 00RAD/SEC   AMPLITUDE= 5.78829D-07   NIT=  4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      KO= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D 00     K3= 1.00000D-02

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.21135223680 01      0.57882942720-06
      1     0.21180918760 01      0.58509301290-06
      2     0.21198952180 01      0.58873250600-06
      3     0.21209051680 01      0.59072797330-06
      4     0.21215524440 01      0.59186241860-06
      5     0.21219679300 01      0.59260231040-06
      6     0.21222618840 01      0.59309245600-06

ND= 1   FREQUENCY= 2.12248D 00RAD/SEC   AMPLITUDE= 5.93446D-07   NIT=  6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      KO= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D 00     K3= 1.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.21224760460 01      0.59344600040-06
      1     0.21673833730 01      0.66628583780-06
      2     0.21958280910 01      0.71140074880-06
      3     0.22128323420 01      0.73721802100-06
      4     0.22248267900 01      0.74964577550-06
      5     0.22276668260 01      0.75994910870-06
      6     0.22301331990 01      0.78620678250-06
      7     0.22267340260 01      0.77067831190-06
      8     0.22303627910 01      0.77142138800-06
      9     0.22313775530 01      0.77085962860-06

ND= 1   FREQUENCY= 2.23141D 00RAD/SEC   AMPLITUDE= 7.71332D-07   NIT=  9
```

Figure 1-23a.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.380000D 07 TO= 1.000000D-01 JV= 1.000000D 05 JG= 2.100000D 00
KP= 2.160000D 02 KI= 9.700000D 03 H= 6.000000D 02 KO= 5.758350 03
K1= 1.371020 03 K2= 1.000000D 00 K3= 2.000000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.22314055190 01	0.77133231820-06
1	0.22918969950 01	0.87037847240-06
2	0.23332686360 01	0.93602606470-06
3	0.23674924000 01	0.97474662960-06
4	0.23758376840 01	0.10015089090-05
5	0.23797925870 01	0.10134590340-05
6	0.23822084950 01	0.10216618780-05
7	0.23826721340 01	0.10260210140-05
8	0.23844971350 01	0.10265748260-05
9	0.23847699590 01	0.10279821240-05
10	0.23852289870 01	0.10283023410-05

ND= 1 FREQUENCY= 2.38548D 00RAD/SEC AMPLITUDE= 1.02888D-06 NIT= 10

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.380000D 07 TO= 1.000000D-01 JV= 1.000000D 05 JG= 2.100000D 00
KP= 2.160000D 02 KI= 9.700000D 03 H= 6.000000D 02 KO= 5.758350 03
K1= 1.371020 03 K2= 1.000000D 00 K3= 3.000000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.23854834610 01	0.10288815160-05
1	0.24678013500 01	0.11650479630-05
2	0.25398528850 01	0.12663483620-05
3	0.25578450900 01	0.12923743980-05
4	0.25776367730 01	0.13442010290-05
5	0.25882643090 01	0.13698989180-05
6	0.25948114380 01	0.13843215460-05
7	0.25995488080 01	0.13952501810-05
8	0.26029329430 01	0.14027589710-05
9	0.26056533510 01	0.14103999390-05
10	0.26054242680 01	0.14254998180-05
11	0.26124580090 01	0.14290249370-05
12	0.26151963730 01	0.14293534980-05
13	0.26152382070 01	0.14301389780-05

ND= 1 FREQUENCY= 2.61534D 00RAD/SEC AMPLITUDE= 1.43025D-06 NIT= 13

Figure 1-23b.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.380000 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      KO= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00     K3= 4.00000D-01

   ITERATION      FREQUENCY                AMPLITUDE
       0     0.26153383290 01     0.14302522270-05
       1     0.27491451770 01     0.16576896760-05
       2     0.27155008060 01     0.18331919630-05
       3     0.28568670450 01     0.15921476640-05
       4     0.30181244280 01     0.22431135990-05
       5     0.32362731020 01     0.26300592780-05
       6     0.29855021500 01     0.22525799410-05
       7     0.31768831180 01     0.25675390120-05
       8     0.19141123090 01     0.42841346750-07
       9     0.20833868460 01     0.64302884600-06
      10     0.22882432030 01     0.93574610080-06
      11     0.24575102290 01     0.11954391140-05
      12     0.25991500320 01     0.14282643080-05
      13     0.27197696170 01     0.16358287280-05
      14     0.28244043390 01     0.18209997380-05
      15     0.29220335750 01     0.19938006230-05
      16     0.28804182890 01     0.20716253490-05
      17     0.30153749000 01     0.22590807780-05

      18     0.32966725560 01     0.27652305310-05
      19     0.30052442540 01     0.22179562590-05
      20     0.32077926070 01     0.25743091210-05

 ND= 1   FREQUENCY= 3.20779D 00RAD/SEC   AMPLITUDE= 2.57431D-06   NIT= 20

 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38C00D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      KO= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00     K3= 5.00000D-01

   ITERATION      FREQUENCY                AMPLITUDE
       0     0.32077926070 01     0.25743091210-05
       1     0.96300656000 00    -0.18201961810-05
 1     9.63007D-01    -1.82020D-06

 IHC900I EXECUTION TERMINATING DUE TO ERROR COUNT FOR ERROR NUMBER  261
```

Figure 1-23c.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 5.00000D 00     K3= 1.00000D-03

   ITERATION     FREQUENCY              AMLITUDE
        0     0.44000000000D 01     0.38000000000D-05
        1     0.43622079060D 01     0.37386218260D-05
        2     0.43666269860D 01     0.37703966090D-05
        3     0.43713786550D 01     0.37966647630D-05
        4     0.43713793860D 01     0.37969710200D-05

ND= 1   FREQUENCY= 4.37138D 00RAD/SEC   AMPLITUDE= 3.79739D-06   NIT=  4


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03    K2= 5.00000D 00     K3= 1.00000D-02

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.43713786550D 01     0.37973907150D-05
        1     0.44394497190D 01     0.39227716410D-05
        2     0.43948472930D 01     0.38066764070D-05
        3     0.43516261430D 01     0.37150209610D-05
        4     0.43081083300D 01     0.36006188890D-05
        5     0.43260200530D 01     0.36707418870D-05
        6     0.43287943910D 01     0.36843250610D-05
        7     0.43281356180D 01     0.36125923910D-05
        8     0.43273652480D 01     0.36467079450D-05
        9     0.43371450590D 01     0.36940378420D-05
       10     0.43406969900D 01     0.37153328880D-05
       11     0.43406655990D 01     0.37171261240D-05

ND= 1   FREQUENCY= 4.34039D 00RAD/SEC   AMPLITUDE= 3.71822D-06   NIT= 11
```

Figure 1-24a.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 5.00000D 00     K3= 1.00000D-01

  ITERATION    FREQUENCY              AMPLITUDE
      0     0.43403937410 01     0.37182237260-05
      1     0.41900859910 01     0.33533919900-05
      2     0.40443154910 01     0.29845729330-05
      3     0.37477095790 01     0.23003378820-05
      4     0.38849551590 01     0.25663780930-05
      5     0.38947926520 01     0.25837072840-05
      6     0.38995721810 01     0.26620491620-05
      7     0.38550207590 01     0.25323411290-05
      8     0.37215277100 01     0.22387643050-05
      9     0.38073527030 01    0.23943328210-05
     10     0.38202025160 01     0.24110788400-05
     11     0.38268549270 01     0.24182133250-05
     12     0.38335051680 01     0.24415333350-05
     13     0.38288367380 01     0.24940320420-05
     14     0.38108071390 01     0.24237090290-05
     15     0.44787694210 01     0.37454161460-05
     16     0.43024043330 01     0.34347071180-05
     17     0.41558981880 01     0.31341721960-05
     18     0.40029479160 01     0.27936375190-05
     19     0.66312788840 01     0.94354327410-05
     20     0.54608032210 01     0.71218484800-05

ND= 1   FREQUENCY= 5.46080D 00RAD/SEC   AMPLITUDE= 7.12185D-06   NIT= 20


LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 5.00000D 00     K3= 2.00000D-01

  ITERATION    FREQUENCY              AMPLITUDE
      0     0.54608032210 01     0.71218484800-05
      1     0.47744974610 01     0.49948179690-05
      2     0.43511513670 01     0.37778461260-05
      3     0.39833271330 01     0.28255197820-05
      4     0.26179854860 01    -0.39573358500-06
 1    2.61799D 00   -3.95734D-07
      5     0.10000002810 01     0.99999972110 00
      6     0.10000000000 01     0.10000000000 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT=  6
```

Figure 1-24b.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 5.00000D 00      K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
        0     0.33000000000D 01     0.13000000000D-05
        1     0.32814815920 01      0.12448820280-05
        2     0.32913200540 01      0.12529977370-05
        3     0.32931224190 01      0.12526203820-05
        4     0.32932847900 01      0.12542029590-05
        5     0.32936025550 01      0.12543762320-05

ND= 1   FREQUENCY= 3.29381D 00RAD/SEC   AMPLITUDE= 1.25492D-06   NIT=  5


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 5.00000D 00      K3= 1.00000D-02

   ITERATION     FREQUENY               AMPLITUDE
        0     0.32938109830 01      0.12549239470-05
        1     0.33017721750 01      0.12680766010-05
        2     0.33064941970 01      0.12770117840-05
        3     0.33072273510 01      0.12935582940-05
        4     0.33156357810 01      0.13005530840-05
        5     0.33161922980 01      0.13013796060-05

ND= 1   FREQUENCY= 3.31660D 00RAD/SEC   AMPLITUDE= 1.30238D-06   NIT=  5
```

Figure 1-25a.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 5.00000D 00     K3= 1.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.3316597982D 01     0.1302379026D-05
      1     0.3413836922D 01     0.1504480331D-05
      2     0.3497137166D 01     0.1675209538D-05
      3     0.3650983247D 01     0.1863445424D-05
      4     0.3822585581D 01     0.2333538259D-05
      5     0.4025859796D 01     0.2869226076D-05
      6     0.4306719963D 01     0.3600958328D-05
      7     0.4163884584D 01     0.3293571013D-05
      8     0.4010876222D 01     0.2903848972D-05
      9     0.3552083028D 01     0.1874565879D-05
     10     0.3642164398D 01     0.2031512541D-05
     11     0.3688151376D 01     0.2110027783D-05
     12     0.3705009393D 01     0.2138959098D-05
     13     0.3711639221D 01     0.2090685945D-05
     14     0.3790756437D 01     0.2319209448D-05
     15     0.3810400446D 01     0.2385603934D-05
     16     0.3814455882D 01     0.2511116680D-05
     17     0.3993908443D 01     0.2866222875D-05
     18     0.3631616890D 01     0.2051966841D-05
     19     0.3705705816D 01     0.2176188941D-05
     20     0.3752088065D 01     0.2253631425D-05

ND= 1   FREQUENCY= 3.75209D 00RAD/SEC   AMPLITUDE= 2.25363D+06   NIT= 20


LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 5.00000D 00     K3= 2.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0     0.3752088065D 01     0.2253631425D-05
      1     0.4209056132D 01     0.3272927164D-05
      2     0.3834035192D 01     0.2560546135D-05
      3     0.5380608385D 01     0.6152138662D-05
      4     0.4758738637D 01     0.4678815369D-05
      5     0.4342781489D 01     0.3681723091D-05
      6     0.3980004715D 01     0.2815716924D-05
      7     0.2544538760D 01    -0.5643857587D-06
  1    2.54454D 00   -5.64386D-07
      8     0.9999998771D 00     0.1000000125D 01
      9     0.1000000000D 01     0.1000000000D 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT=  9
```

Figure 1-25b.

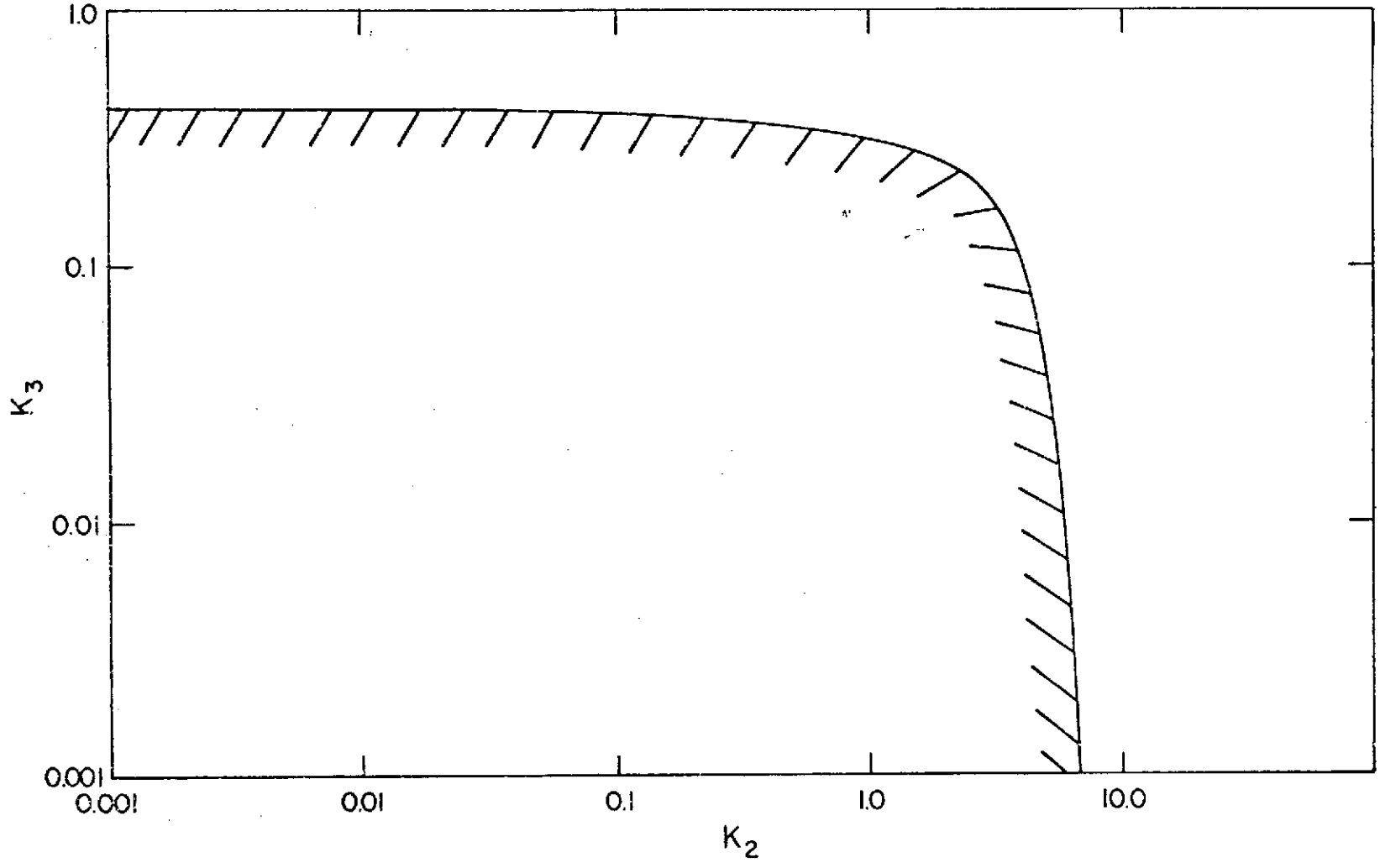


Figure 1-26. Region of self-sustained oscillations in the K_2 - K_3 plane for the two-axis LST system.

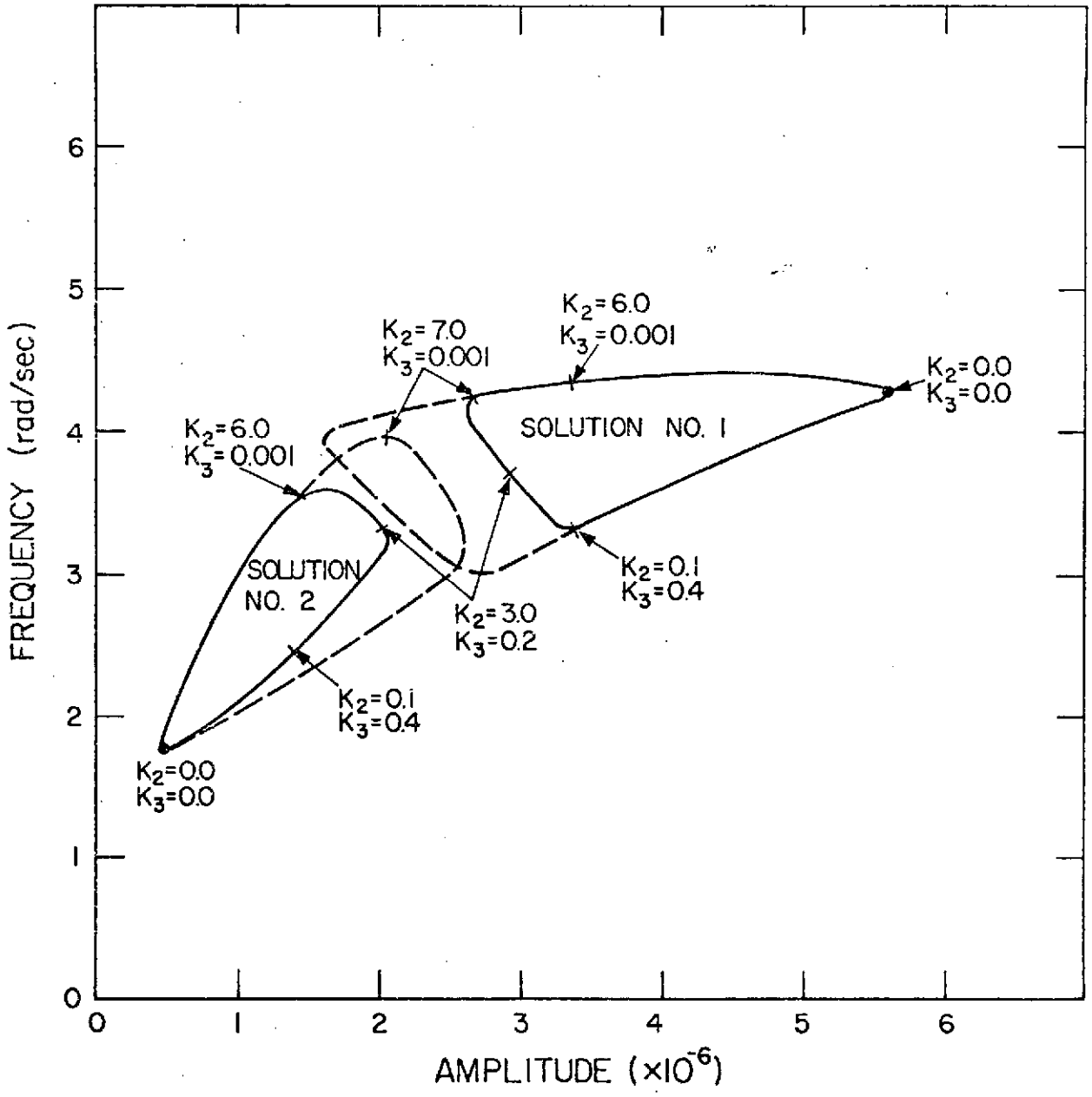


Figure 1-27. Region of possible solution points for the two-axis LST system.

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